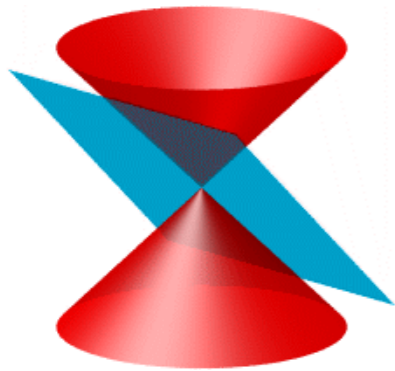
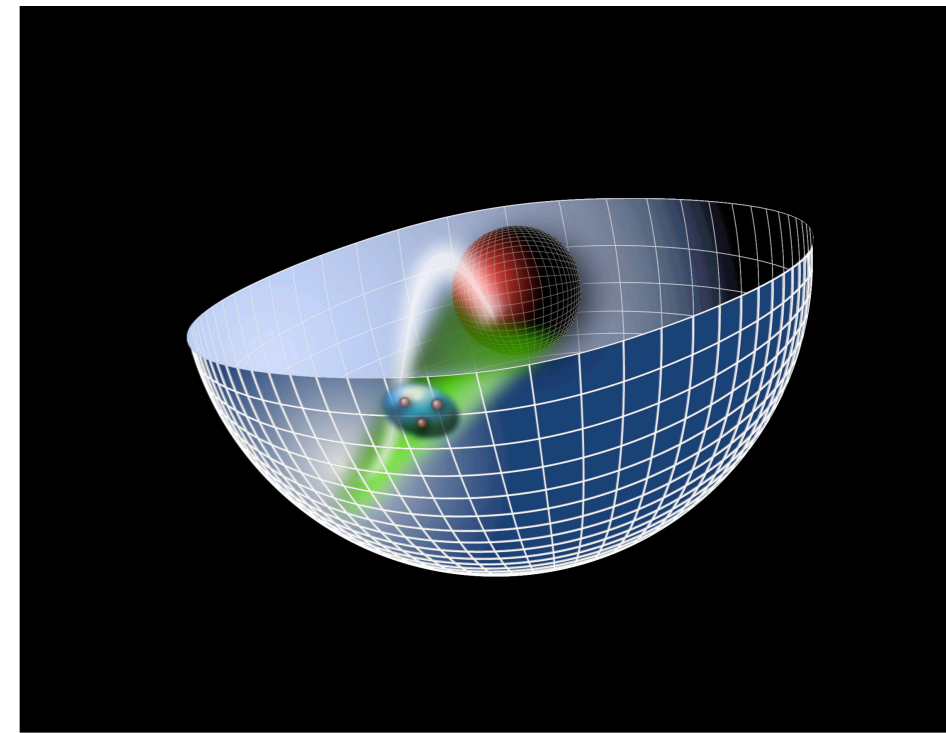
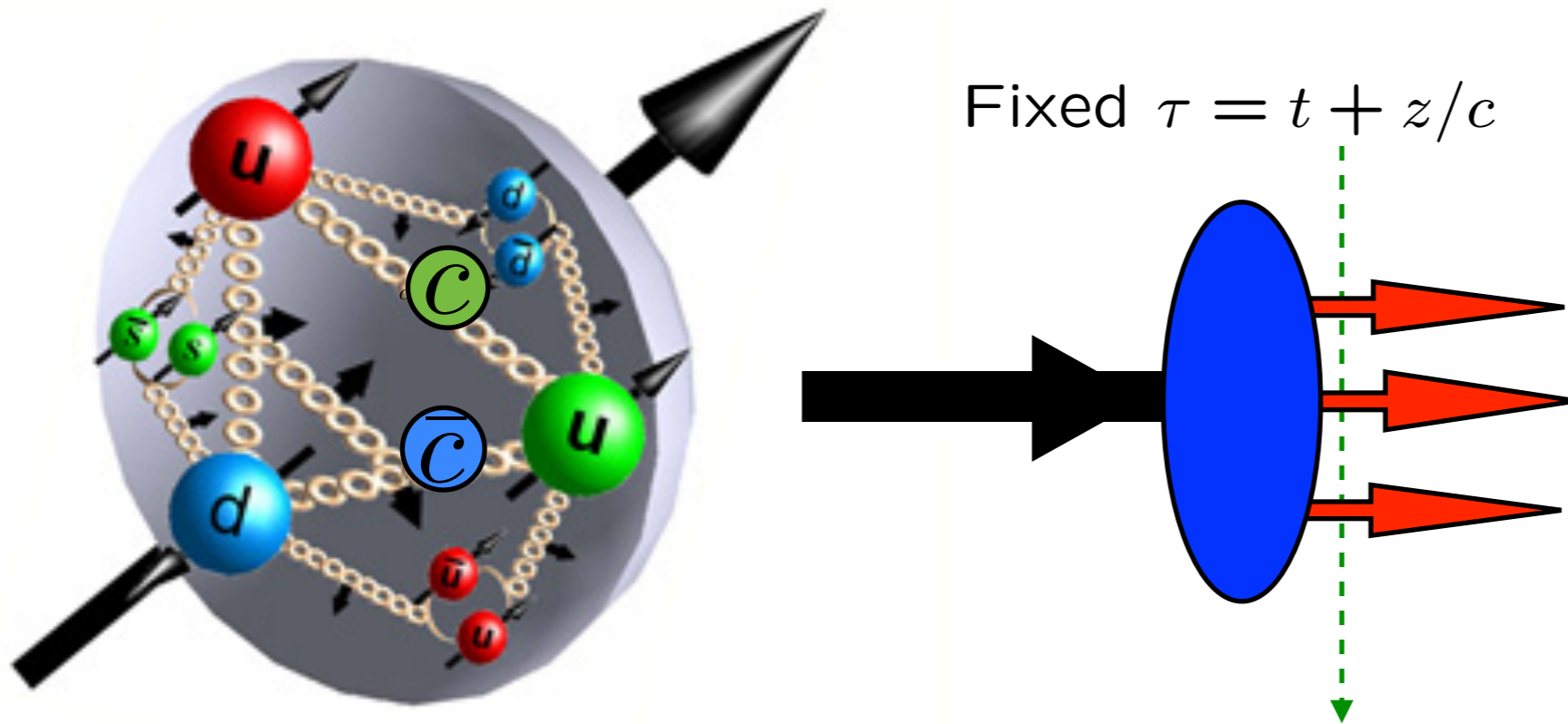


New Perspectives for Hadron Physics and the Cosmological Constant Problem



What Comes Beyond the Standard Model?



*Bled, Slovenia
July 17, 2015*

Stan Brodsky



Goal: An analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **Confinement in QCD -- What is the analytic form of the confining interaction?**
- **What sets the QCD mass scale?**
- **QCD Running Coupling at all scales**
- **Hadron Spectroscopy-Regge Trajectories**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Insights into QCD Condensates**
- **Chiral Symmetry**
- **Systematically improvable**

Slovenia
July 2015



*New Perspectives for Hadron Physics
and the Cosmological Constant*

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics quark kinetic energy +
quark-gluon dynamics quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if $m_q=0$

**Yang Mills Gauge Principle: Color
Rotation and Phase Invariance at
Every Point of Space and Time**

**Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement**

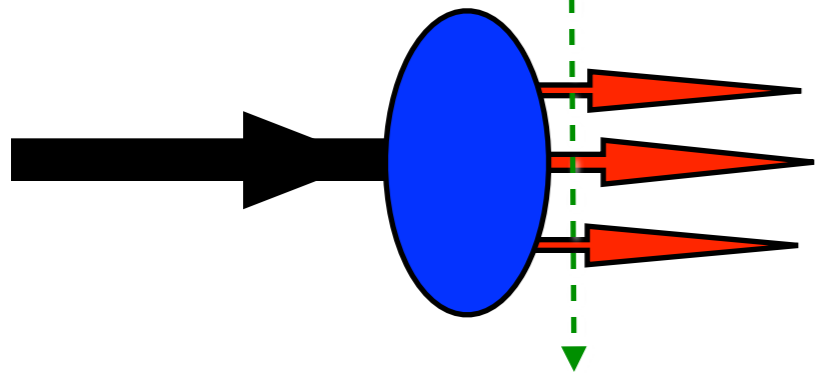
QCD Mass Scale from Confinement not Explicit

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

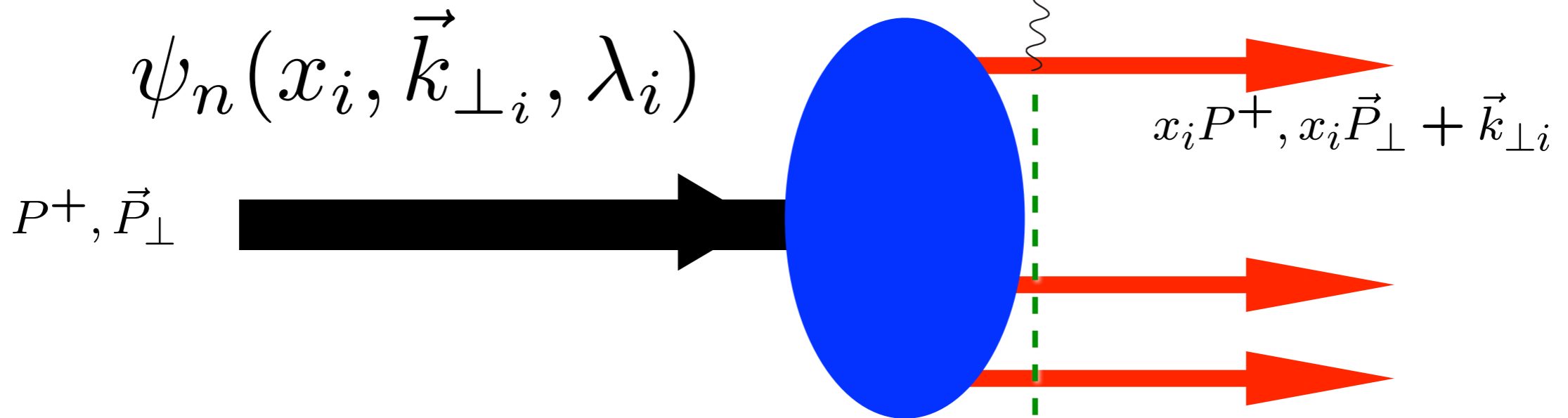
$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

Eigenstate -- independent of τ

$$H_{LF} = P^+ P^- - \vec{P}_\perp^2$$

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$



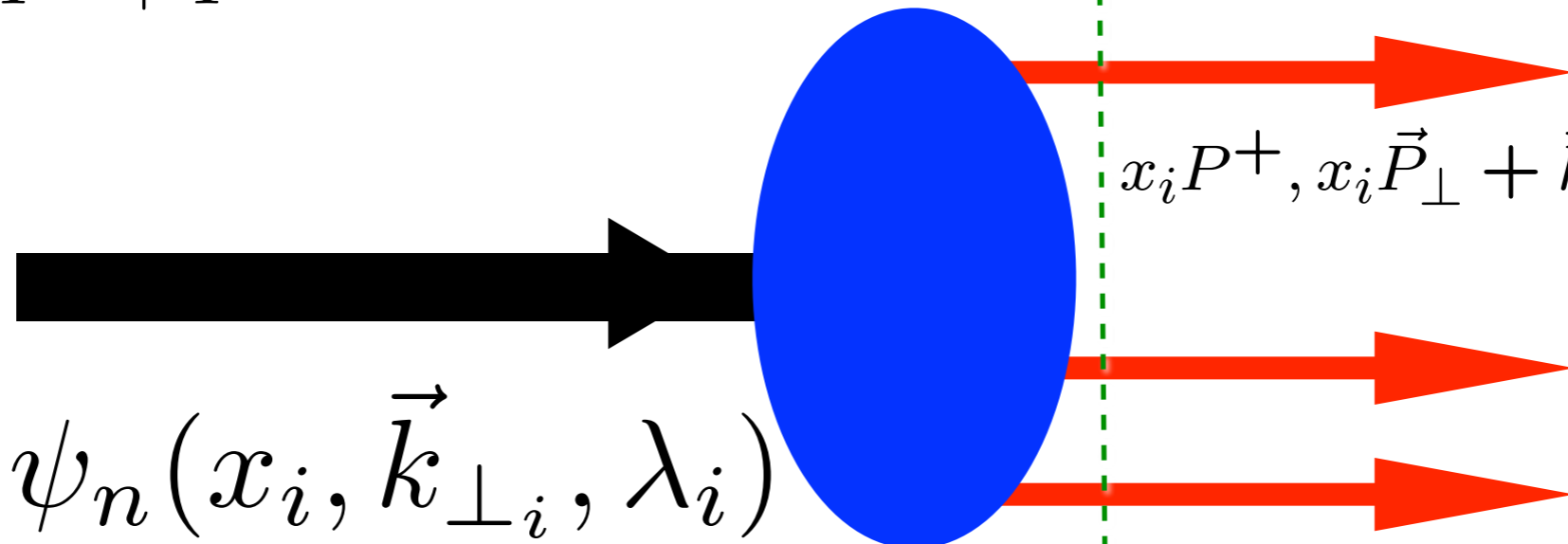
Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$

P^+, \vec{P}_\perp



$$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

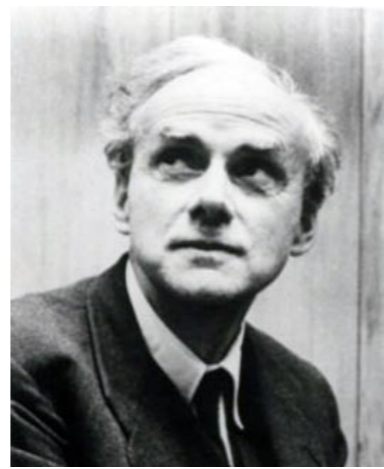
$$\int \psi_{BS}(p, k) dk^- \rightarrow \psi_{LF}$$

$$|p, J_z \rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i \rangle$$

Invariant under boosts! Independent of P^μ

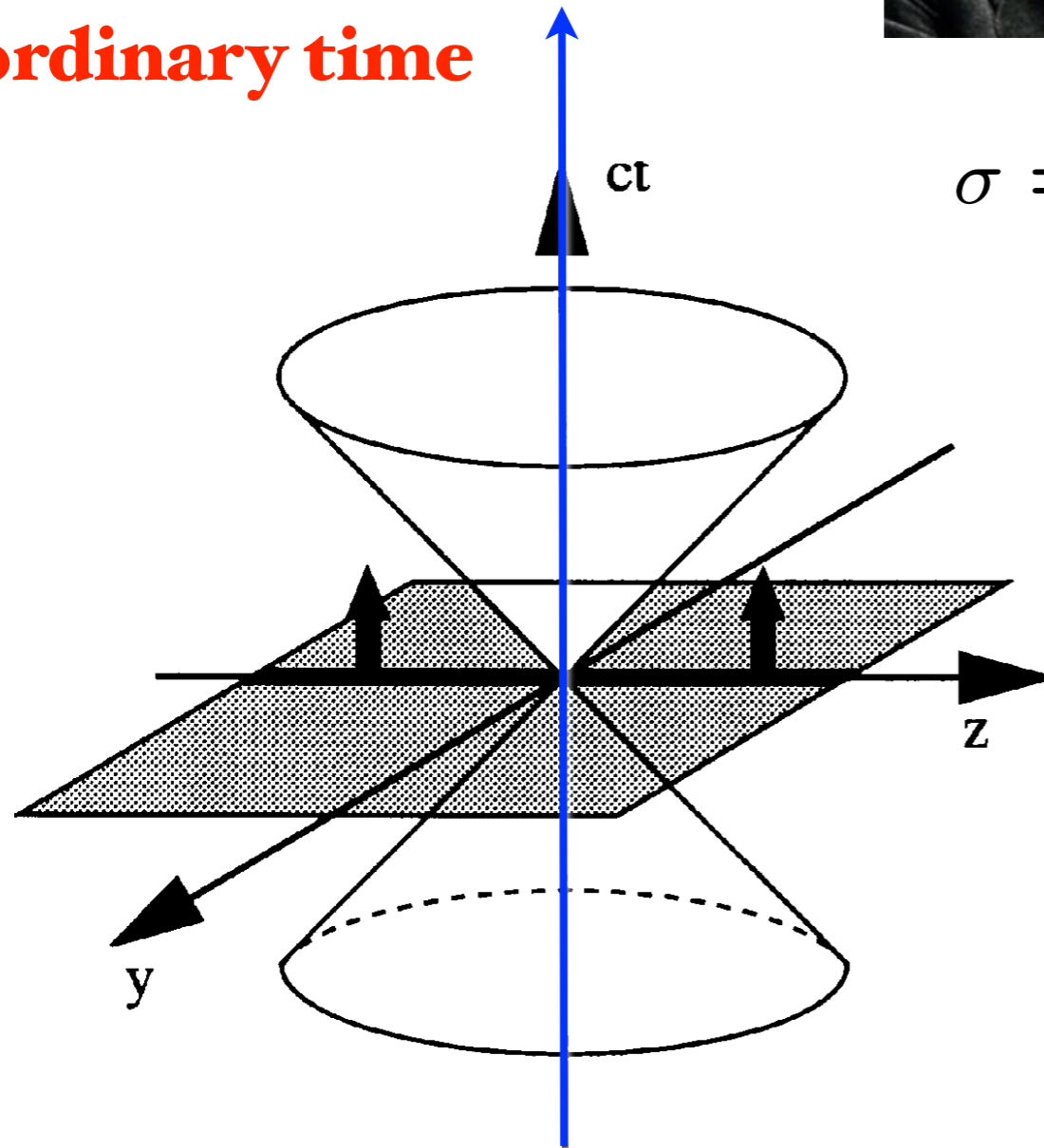
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

*Dirac's Amazing Idea:
The "Front Form"*



**P.A.M Dirac, Rev. Mod. Phys. 21,
392 (1949)**

**Evolve in
ordinary time**

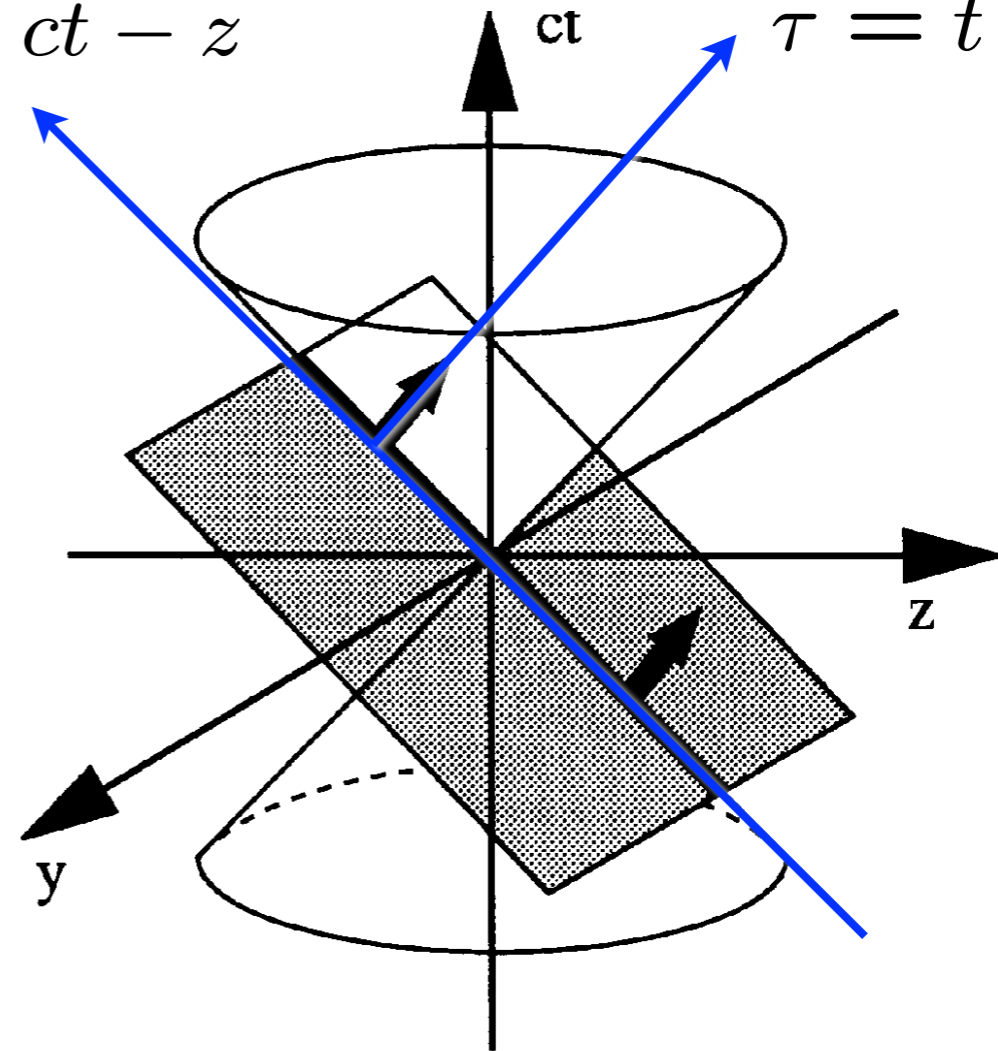


Instant Form

**Evolve in
light-front time!**

$$\sigma = ct - z$$

$$\tau = t + z/c$$



Front Form

Boost Invariant!

$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

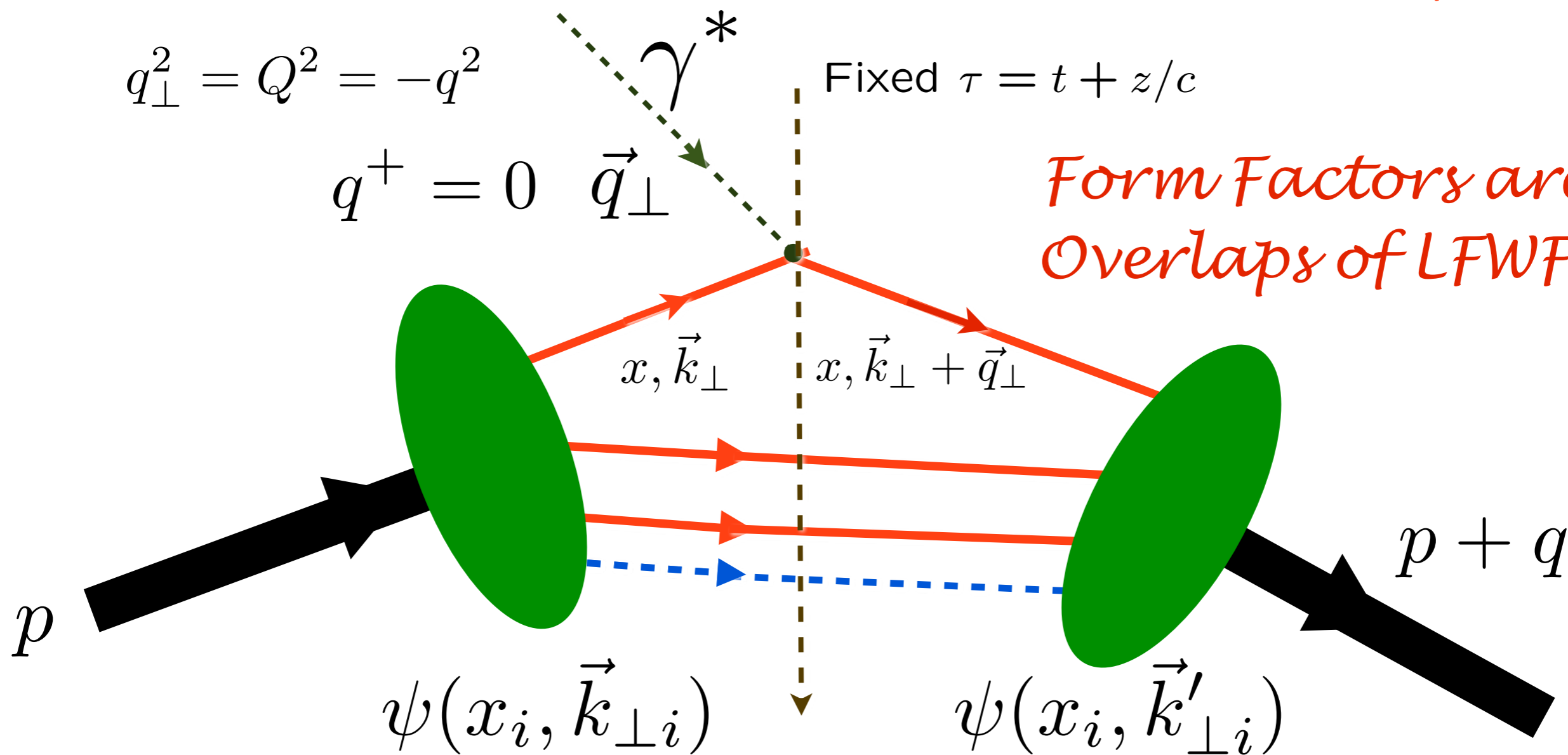
Interaction picture

$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

Fixed $\tau = t + z/c$

Form Factors are Overlaps of LFWFs



$$\psi(x_i, \vec{k}_{\perp i})$$

$$\psi(x_i, \vec{k}'_{\perp i})$$

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

**Drell & Yan, West
Exact LF formula!**

No comparable formula in instant form

Exact LF Formula for Pauli Form Factor

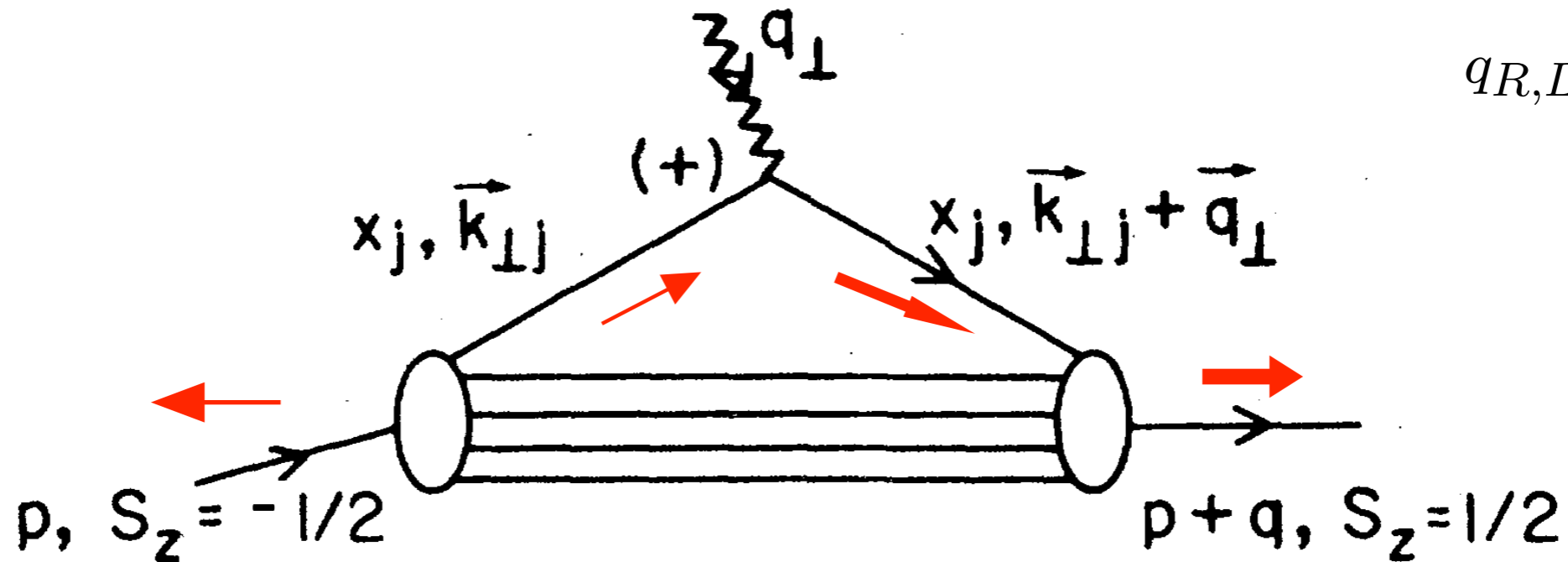
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

Drell, sjb

$$q_{R,L} = q^x \pm iq^y$$



Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Gravitational Form Factors

$$\langle P' | T^{\mu\nu}(0) | P \rangle = \bar{u}(P') \left[A(q^2) \gamma^{(\mu} \bar{P}^{\nu)} + B(q^2) \frac{i}{2M} \bar{P}^{(\mu} \sigma^{\nu)\alpha} q_\alpha + C(q^2) \frac{1}{M} (q^\mu q^\nu - g^{\mu\nu} q^2) \right] u(P) ,$$

where $q^\mu = (P' - P)^\mu$, $\bar{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $a^{(\mu} b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

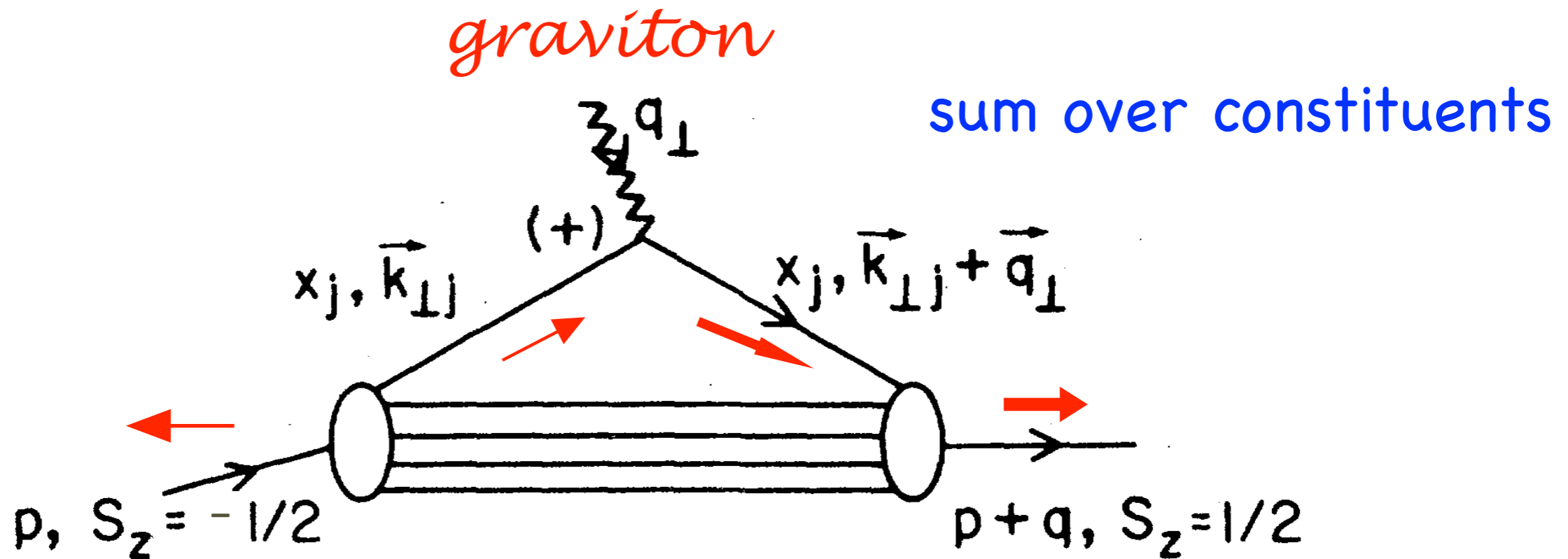
$$\left\langle P + q, \uparrow \left| \frac{T^{++}(0)}{2(P^+)^2} \right| P, \uparrow \right\rangle = A(q^2) ,$$

$$\left\langle P + q, \uparrow \left| \frac{T^{++}(0)}{2(P^+)^2} \right| P, \downarrow \right\rangle = -(q^1 - iq^2) \frac{B(q^2)}{2M} .$$



Vanishing Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem



Hwang, Schmidt, sjb;
Holstein et al

$$B(0) = 0$$

Each Fock State

Angular Momentum on the Light-Front

LC gauge

$A^+=0$

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Glueon orbital angular momentum defined in physical lc gauge

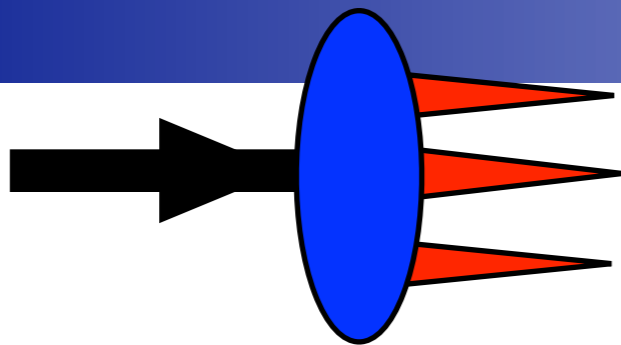
$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right) \quad n-1 \text{ orbital angular momenta}$$

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->

Nonzero quark orbital angular momentum!

pQED: Ma, Hwang, Schmidt, sjb

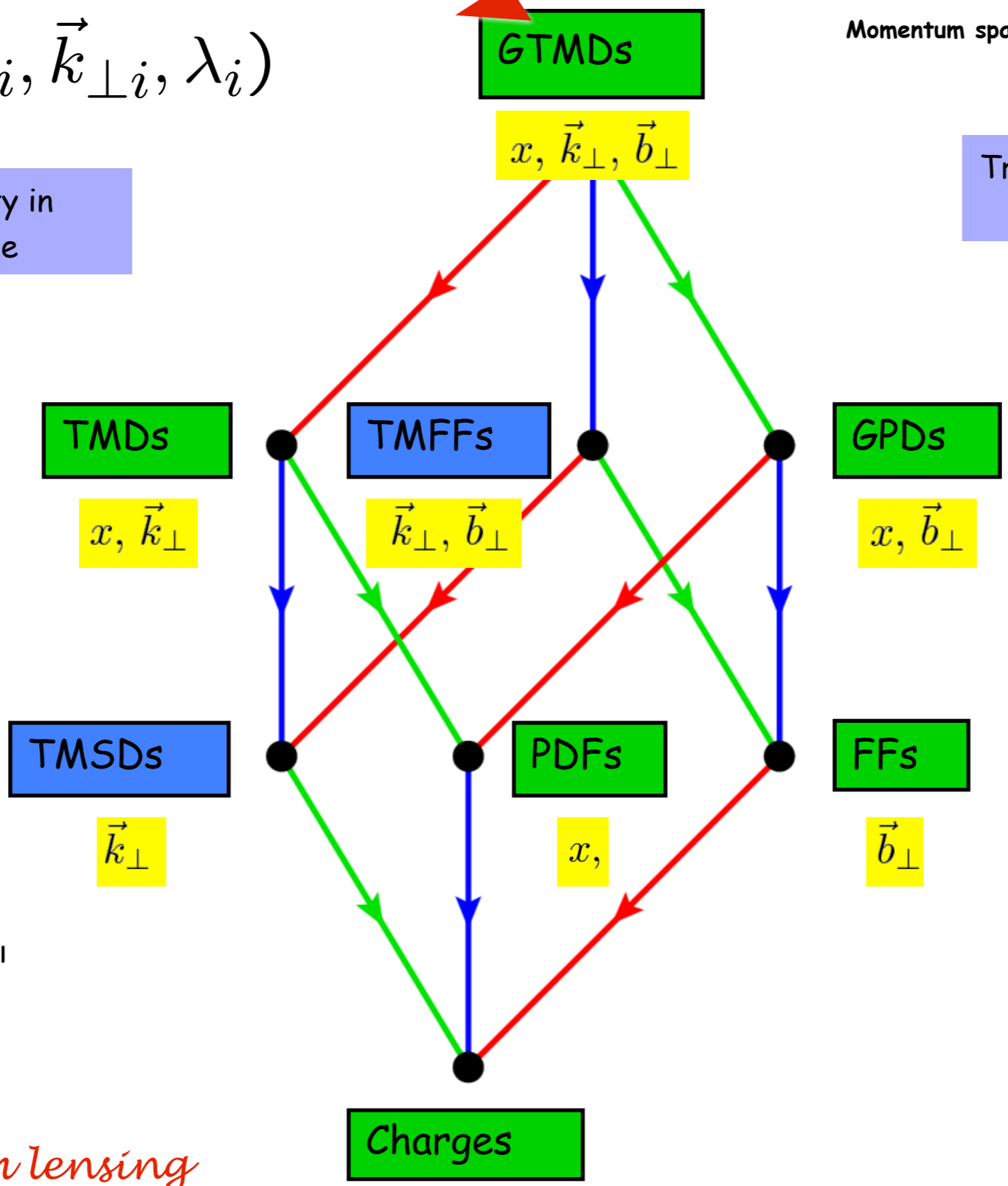


• *Light Front Wavefunctions:*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

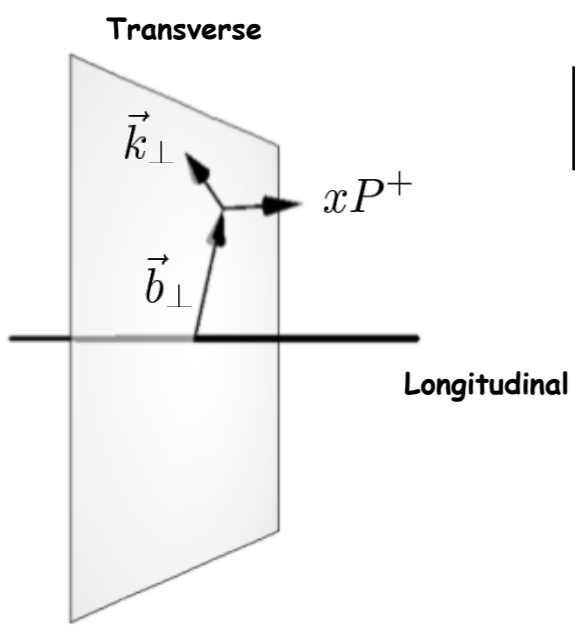
Transverse density in momentum space

Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$
 Transverse density in position space



*Lorce,
Pasquini*

→ $\int d^2 b_{\perp}$
 → $\int dx$
 → $\int d^2 k_{\perp}$



Sivers, T-odd from lensing

Single-spin asymmetries

Leading Twist Sivers Effect

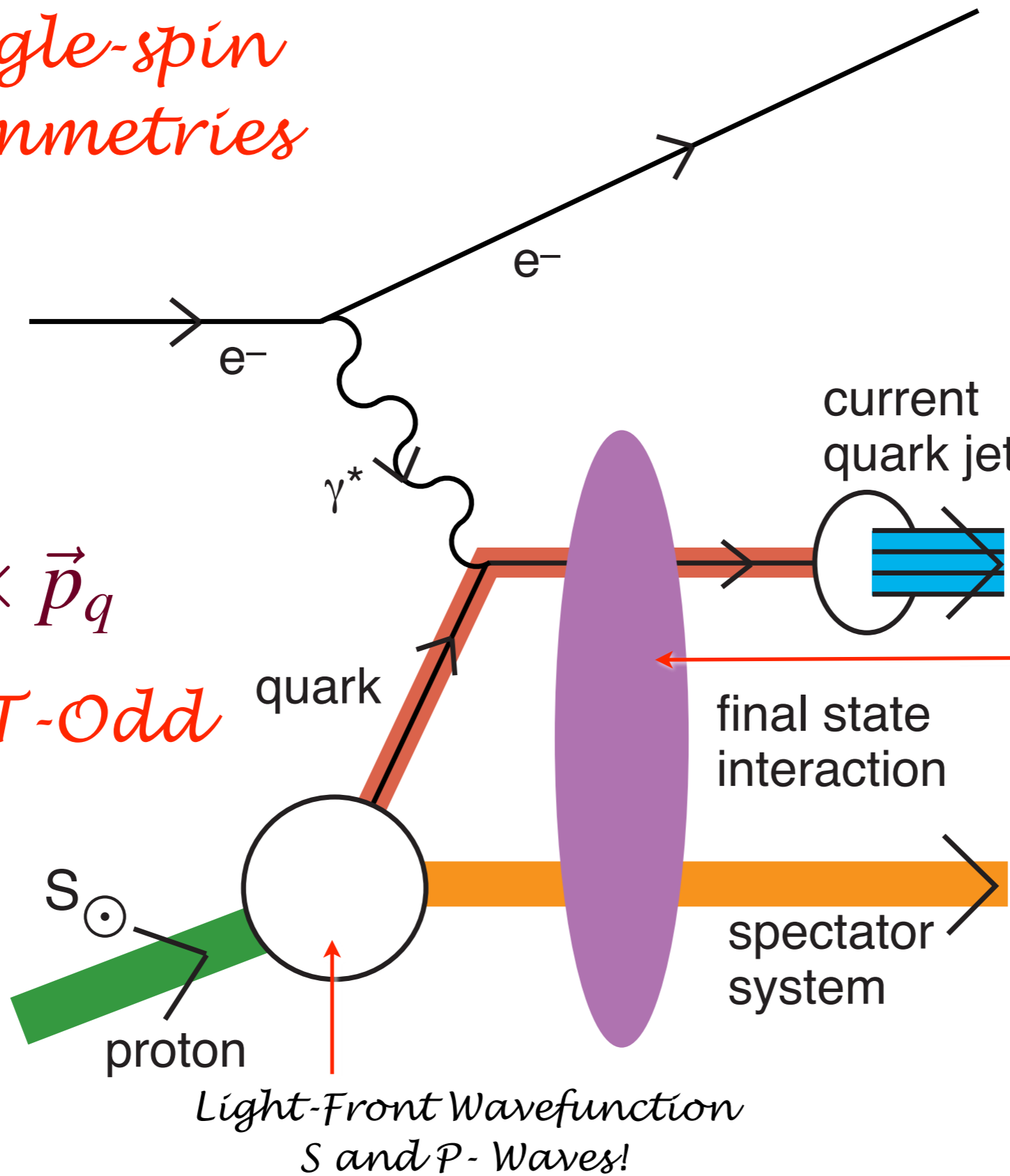
Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

QCD S- and P-Coulomb Phases --Wilson Line

“Lensing Effect”

Leading-Twist Rescattering Violates pQCD Factorization!



$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

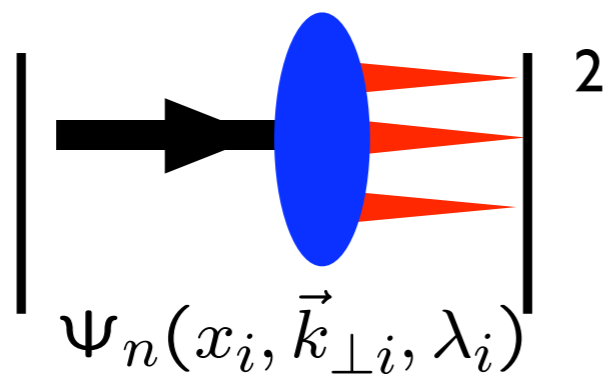
“Lensing” involves soft scales

Sign reversal in DY!

Light-Front Wavefunction S and P-Waves!

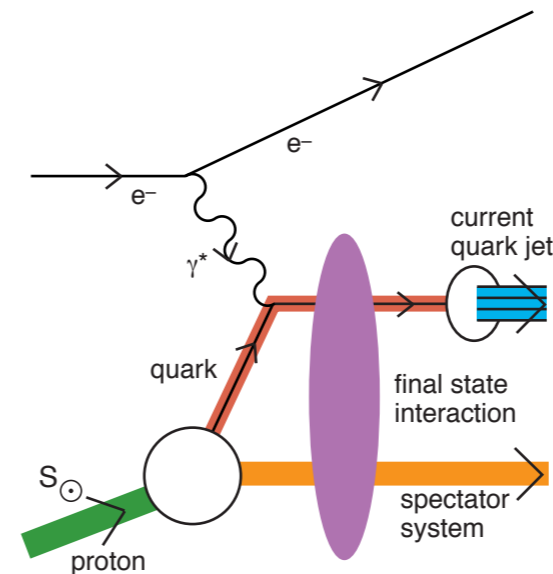
Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and J^z
- DGLAP Evolution; mod. at large x
- No Diffractive DIS

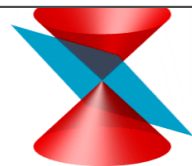


Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS



**Hwang,
Schmidt, sjb,
Mulders, Boer
Qiu, Sterman
Collins, Qiu
Pasquini, Xiao,
Yuan, sjb**



Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

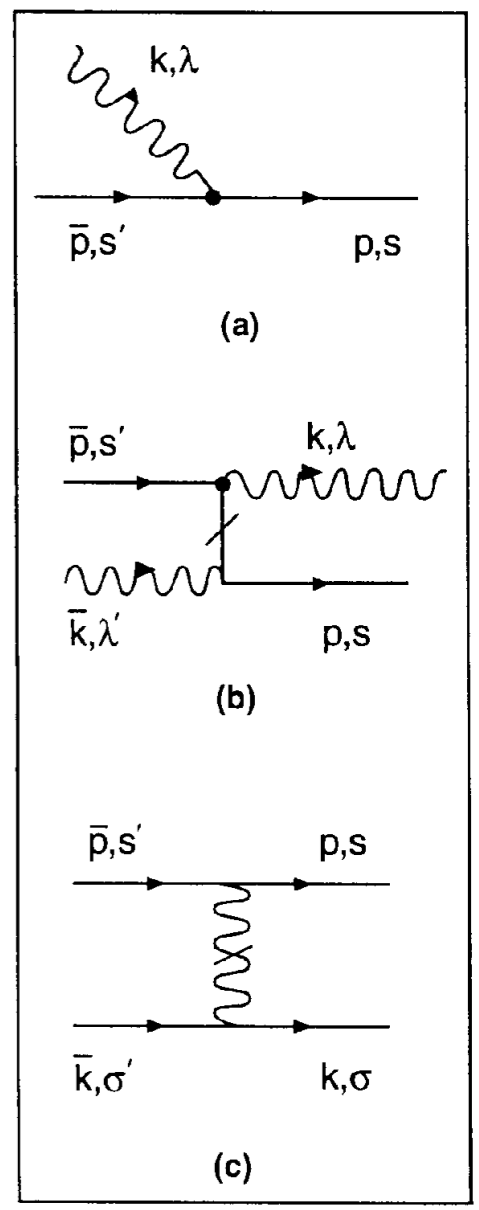
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

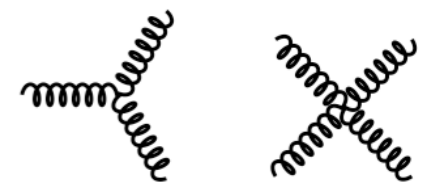
$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass

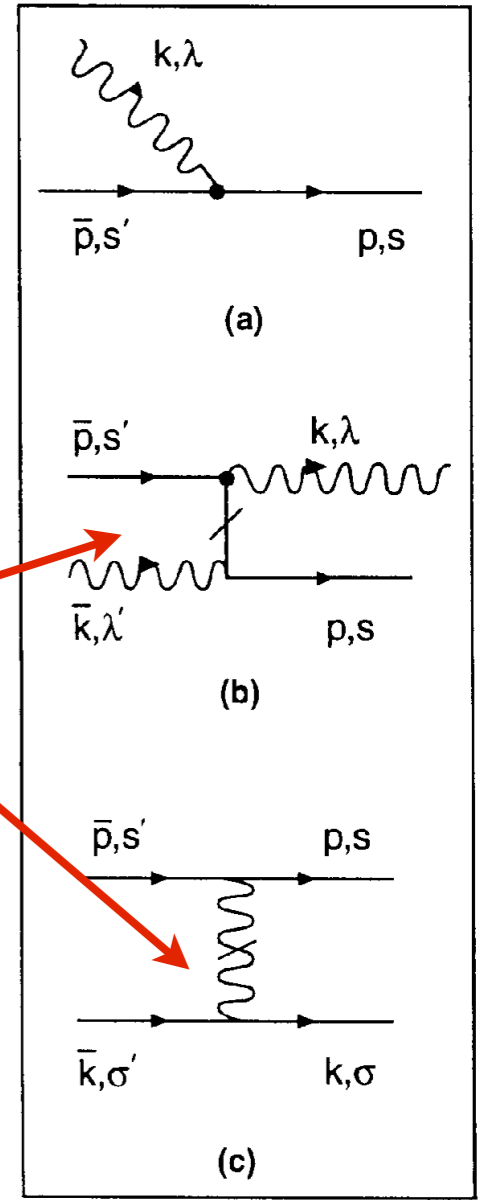
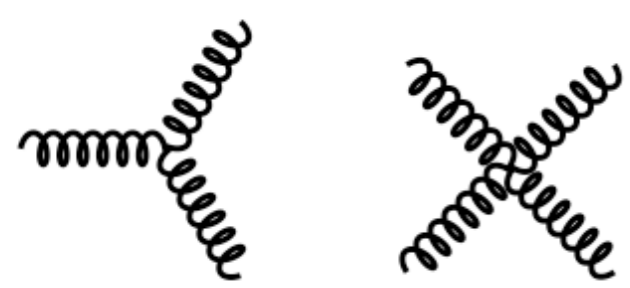


H_{LF}^{int}

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

H_{QCD}^{LF}

$$\begin{aligned} &= \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \psi - A_a^i (i\partial^\perp)^2 A_{ia} \\ &- \frac{1}{2} g^2 \int d^3x \text{Tr} [\tilde{A}^\mu, \tilde{A}^\nu] [\tilde{A}_\mu, \tilde{A}_\nu] \\ &+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \\ &- g^2 \int d^3x \bar{\psi} \gamma^+ \left(\frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \right) \psi \\ &+ g^2 \int d^3x \text{Tr} \left([i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \right) \\ &+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \tilde{A} \frac{\gamma^+}{i\partial^+} \tilde{A} \psi \\ &+ g \int d^3x \bar{\psi} \tilde{A} \psi \\ &+ 2g \int d^3x \text{Tr} (i\partial^\mu \tilde{A}^\nu [\tilde{A}_\mu, \tilde{A}_\nu]) \end{aligned}$$



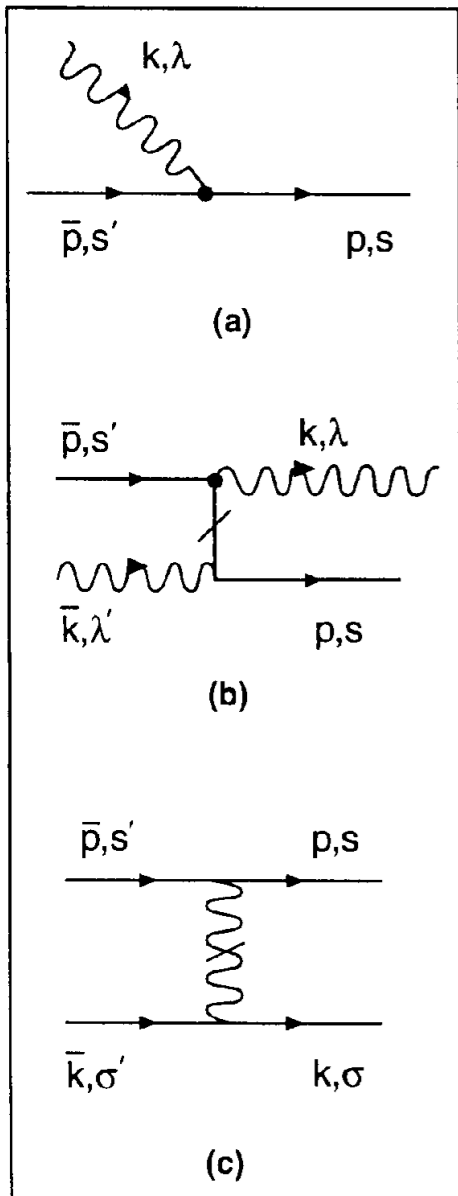
Physical gauge: $A^+ = 0$

Light-Front QCD
Heisenberg Equation

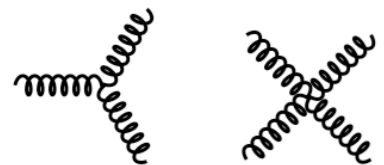
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb



n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 gg g	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gg gg	10 q \bar{q} gg g	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}				
2	gg			
3	q \bar{q} g							
4	q \bar{q} q \bar{q}	
5	gg g
6	q \bar{q} gg								.				.	.
7	q \bar{q} q \bar{q} g
8	q \bar{q} q \bar{q} q \bar{q}			
9	gg gg
10	q \bar{q} gg g
11	q \bar{q} q \bar{q} gg
12	q \bar{q} q \bar{q} q \bar{q} g			
13	q \bar{q} q \bar{q} q \bar{q} q \bar{q}		



Minkowski space; frame-independent; no fermion doubling; no ghosts
trivial vacuum

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

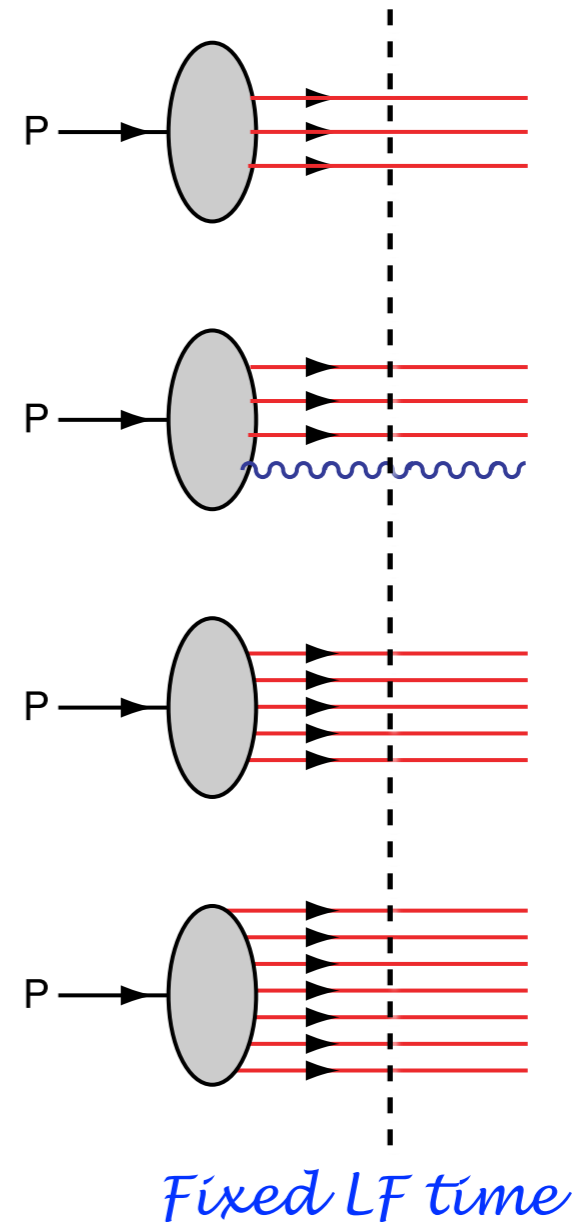
are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



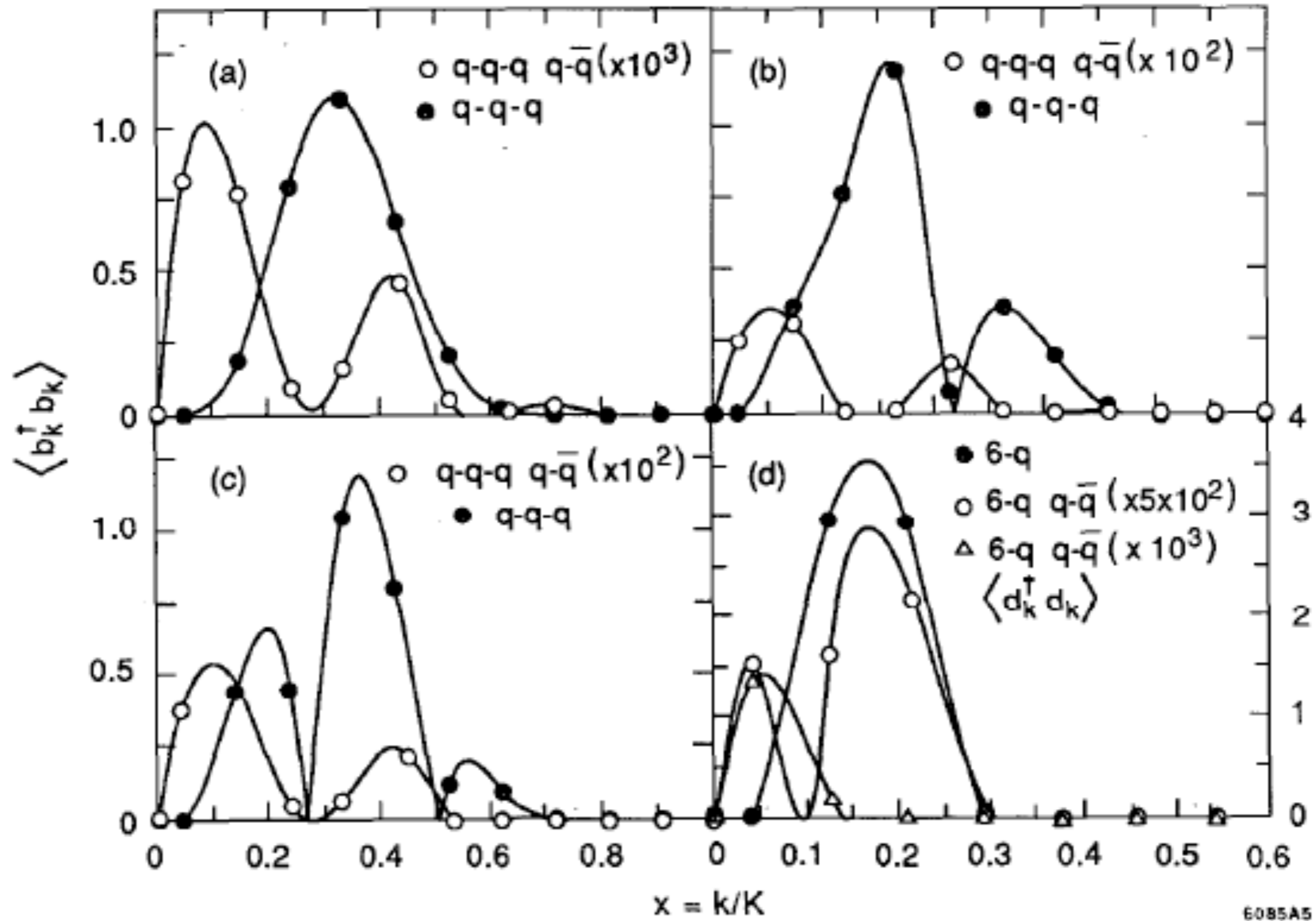
Intrinsic heavy quarks
 $s(x), c(x), b(x)$ at high x !

$\bar{s}(x) \neq s(x)$
 $\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states

BFKL Pomeron

Hidden Color

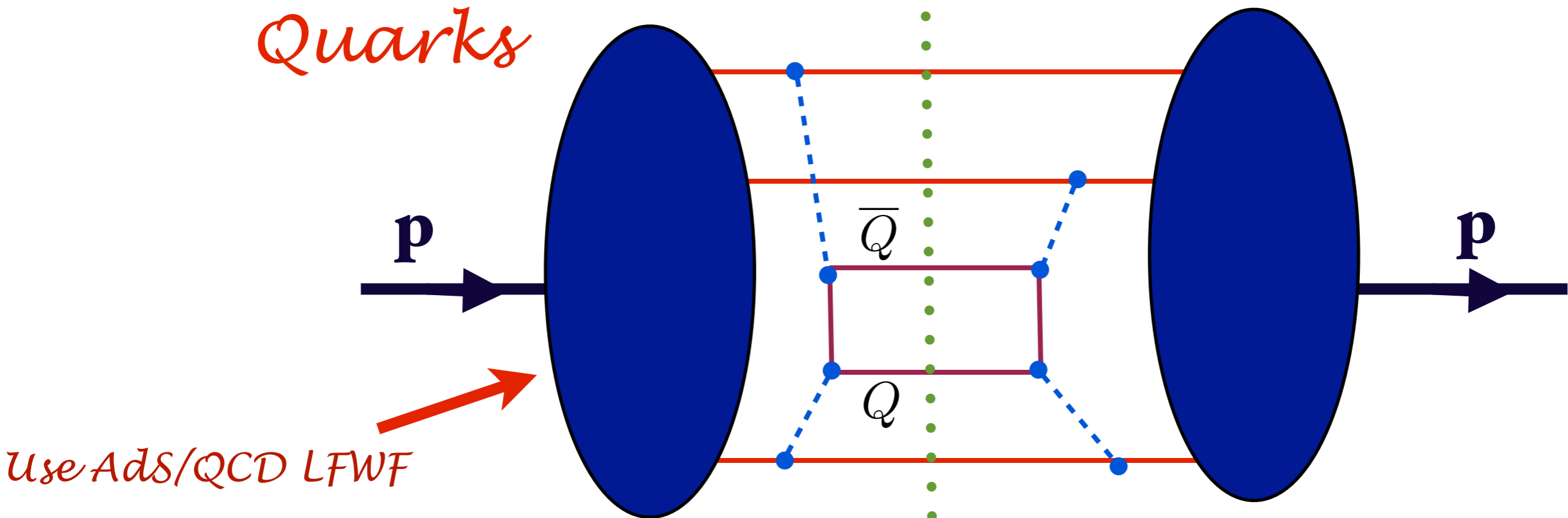


a-c) First three states in $N = 3$ baryon spectrum, $2K=21$. d) First $B = 2$ state.

*Proton Self Energy
Intrinsic Heavy
Quarks*

Fixed LF time

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$



Use AdS/QCD LFWF

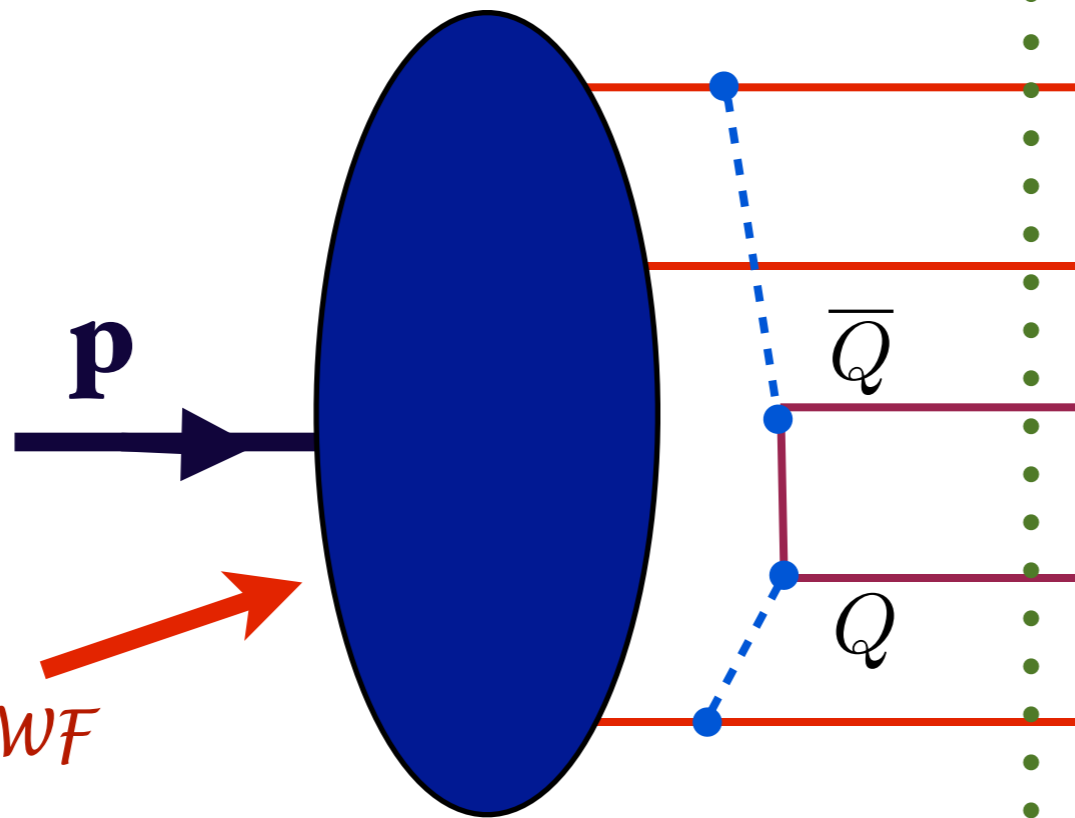
Probability (QED) $\propto \frac{1}{M_{\ell}^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

**Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.**

Fixed LF time

Proton 5-quark Fock State:
Intrinsic Heavy Quarks



QCD predicts
Intrinsic Heavy
Quarks at high x .

Minimal off-shellness

Use AdS/QCD LFWF

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

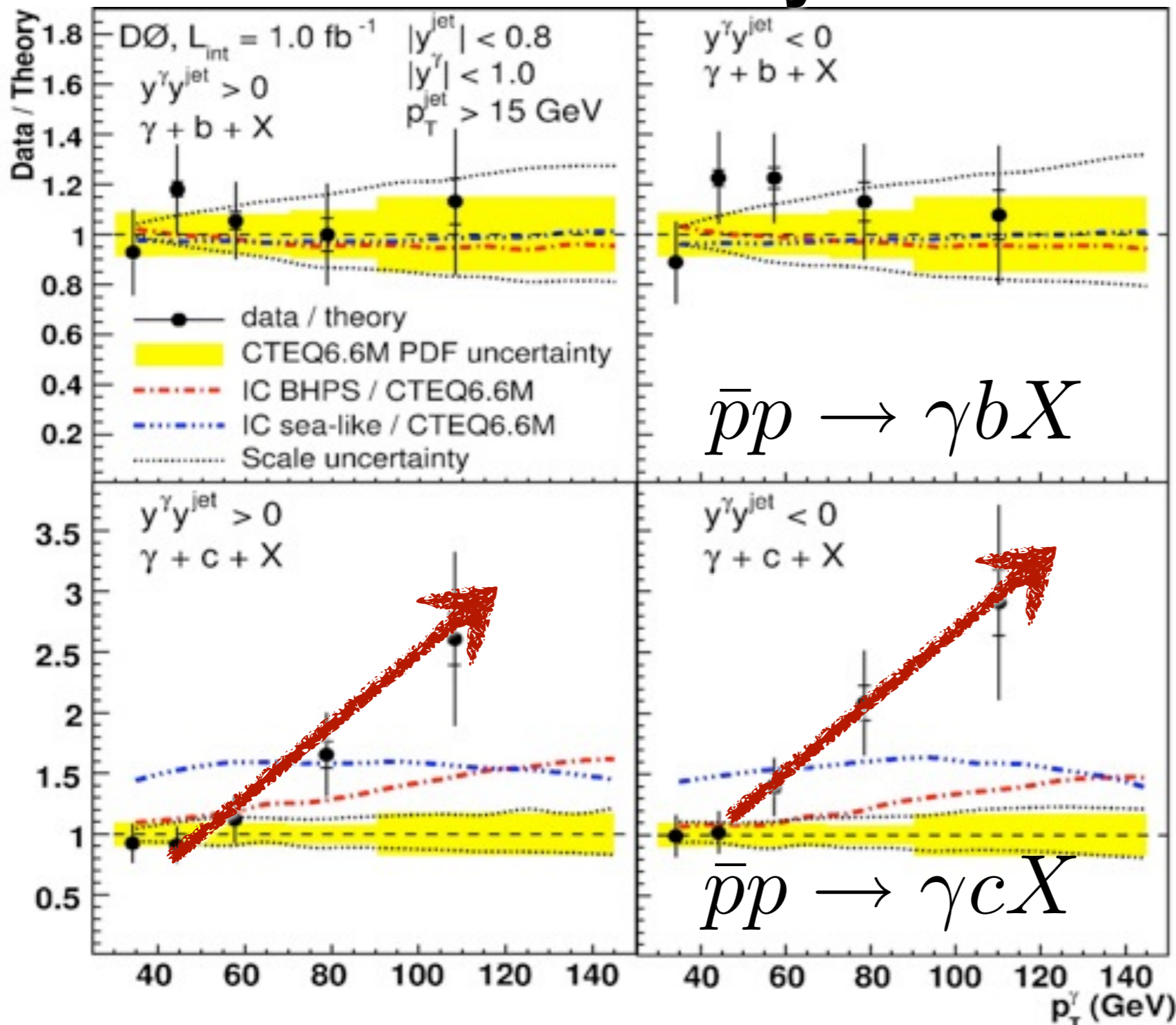
$$\text{Probability (QED)} \propto \frac{1}{M_{\ell}^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections
in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

Data/Theory



$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

**Ratio insensitive
to gluon PDF,
scales**

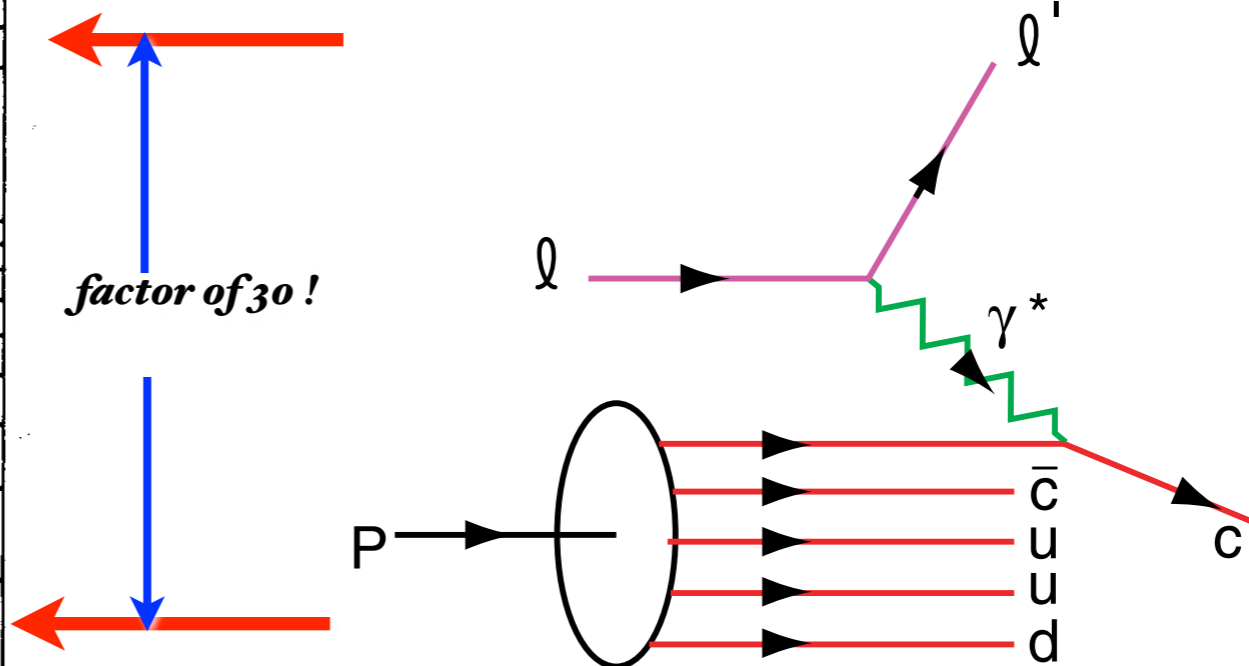
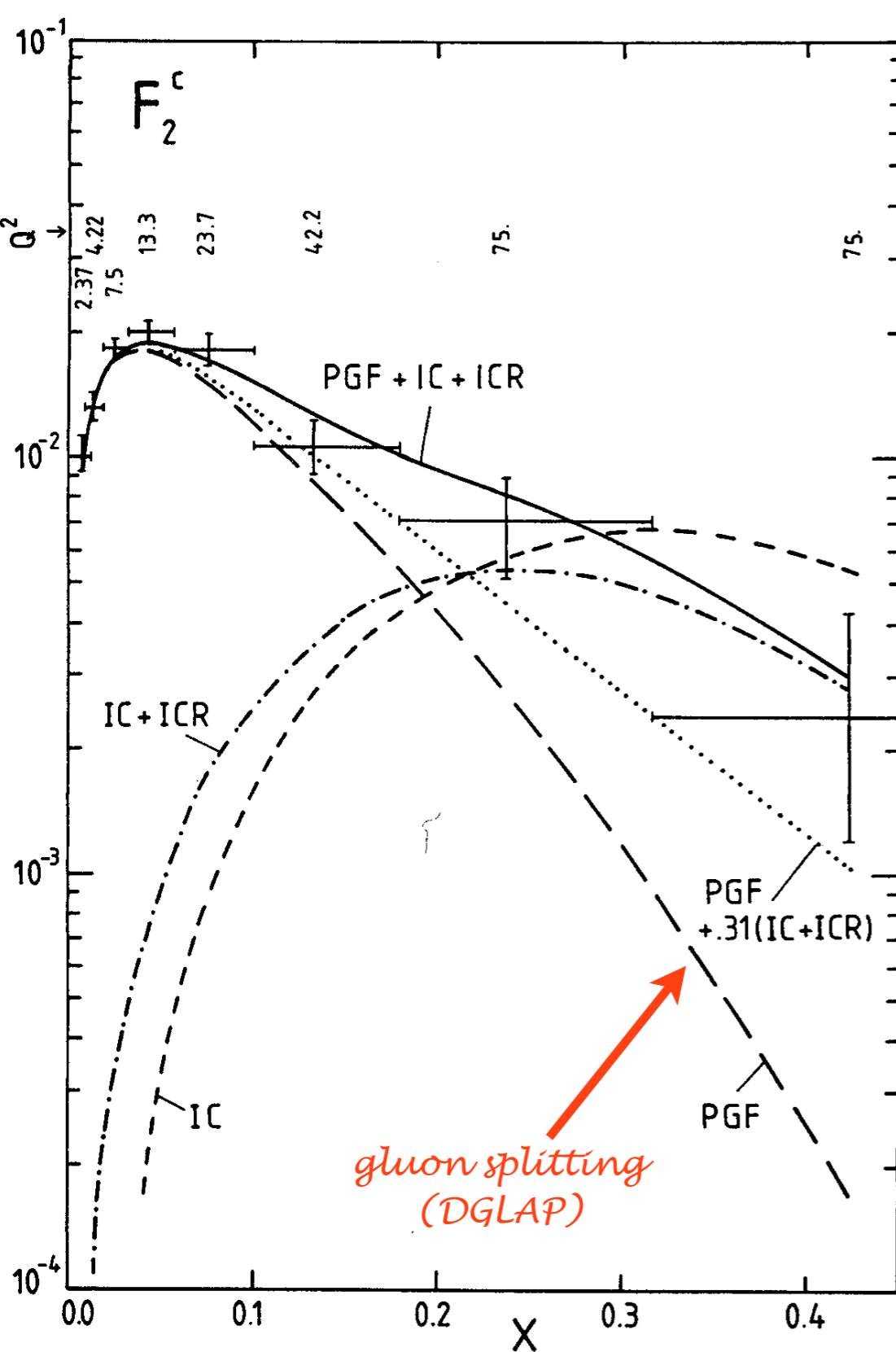
**Signal for significant
IC
at $x > 0.1$**

Consistent with EMC measurement of charm structure function at high x

Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm Hoyer, Peterson, Sakai, sjb

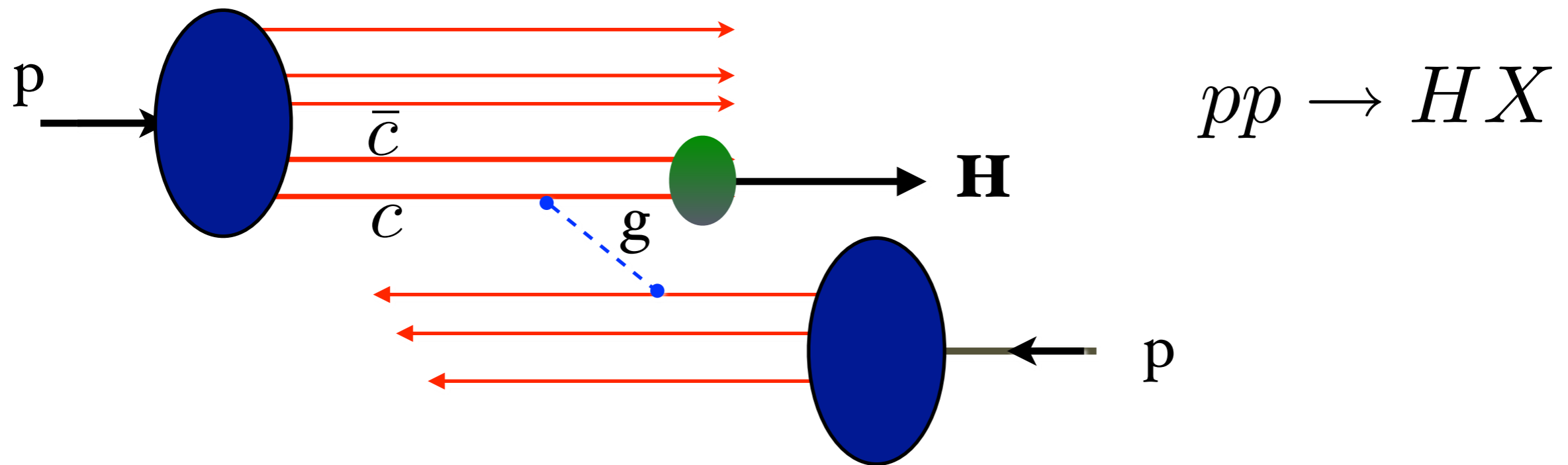


DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

*Intrinsic Charm Mechanism for Inclusive
High- x_F Higgs Production*



Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

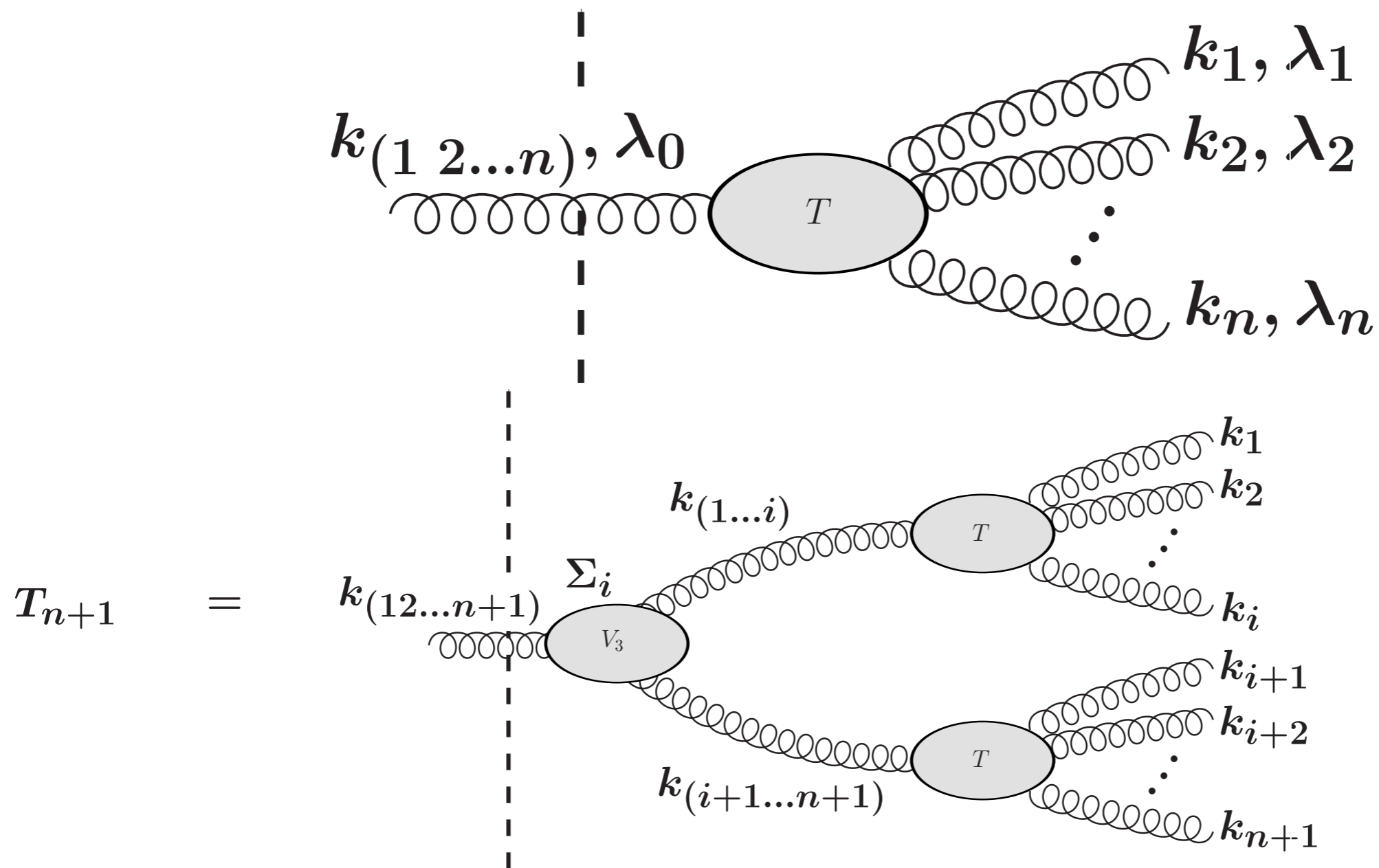
AFTER: Higgs production at threshold!

JLab: Charm production near threshold!

Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

Cluster Decomposition Theorem for relativistic systems: **C. Ji & sjb**



Parke-Taylor amplitudes reflect LF angular momentum conservation

$$\langle ij \rangle = \sqrt{z_i z_j} \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) =$$

Need a First Approximation to QCD

*Comparable in simplicity to
Schrödinger Theory in Atomic Physics*

Relativistic, Frame-Independent, Color-Confining

H_{QED}

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

$$\left[-\frac{\Delta^2}{2m_{red}} + V_{eff}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{red}} \frac{d^2}{dr^2} + \frac{1}{2m_{red}} \frac{\ell(\ell+1)}{r^2} + V_{eff}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Semiclassical first approximation to QED

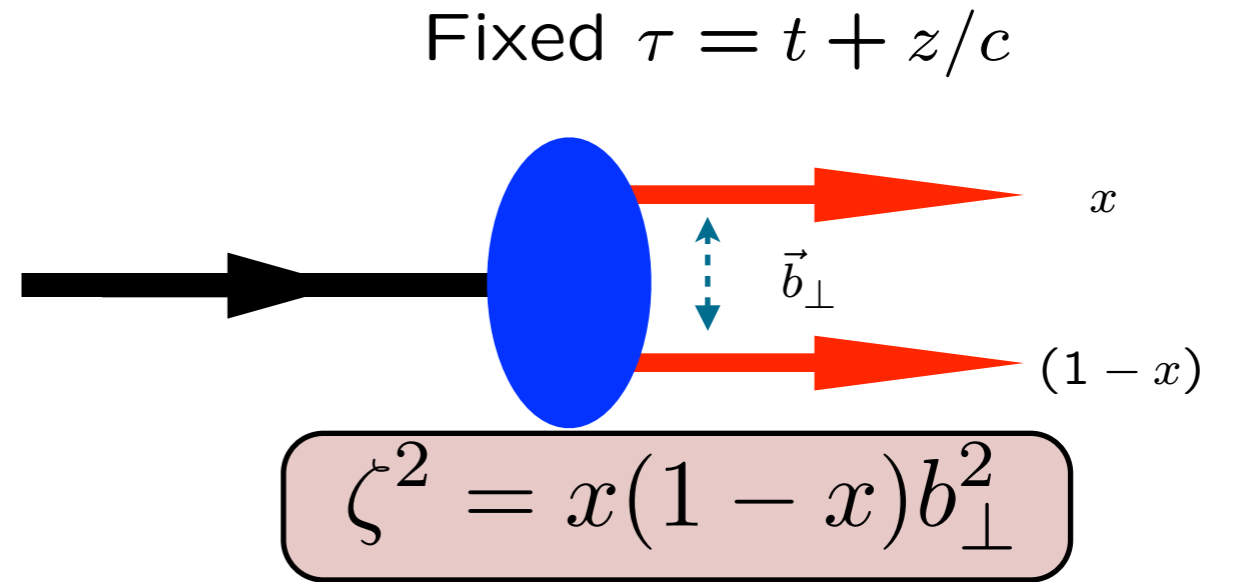
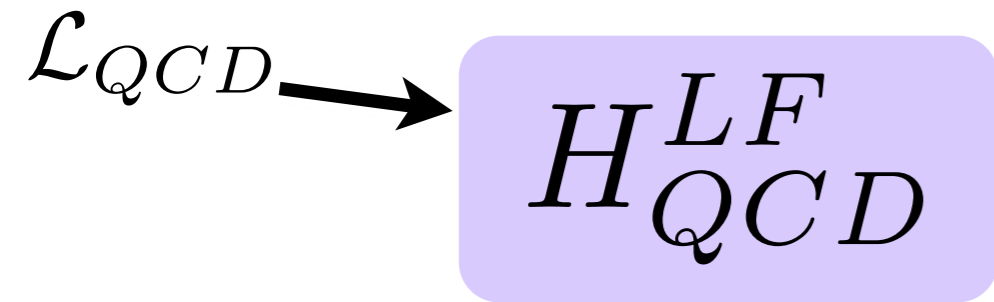


Coulomb potential

Bohr Spectrum

Schrödinger Eq.

Light-Front QCD



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}_{\perp}^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_{\perp}) = M^2 \psi_{LF}(x, \vec{k}_{\perp})$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

$$m_q = 0$$

AdS/QCD:

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

Sums an infinite # diagrams

Light-Front Schrödinger Equation

G. de Teramond, sjb

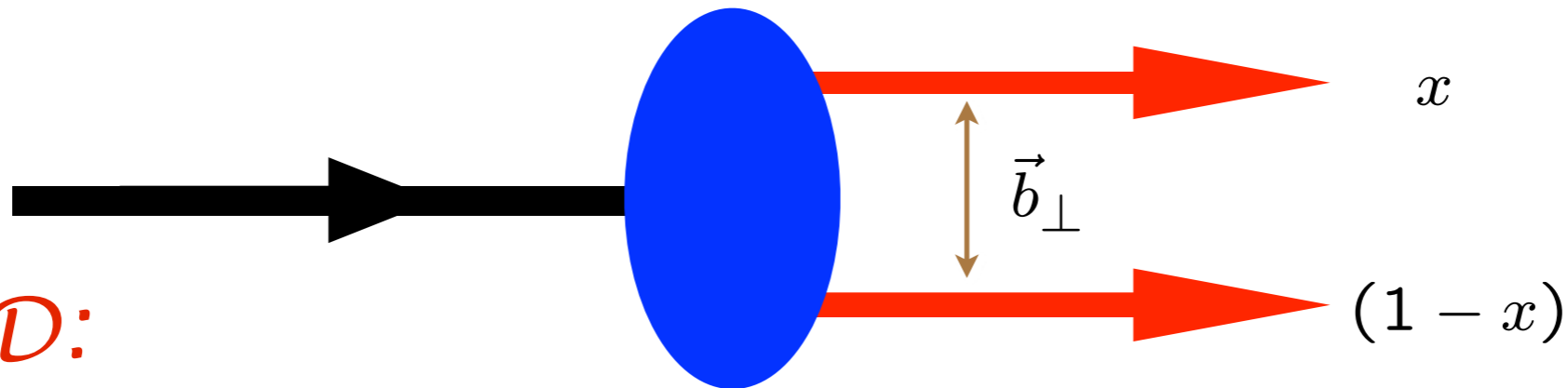
Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

$$m_q \sim 0$$

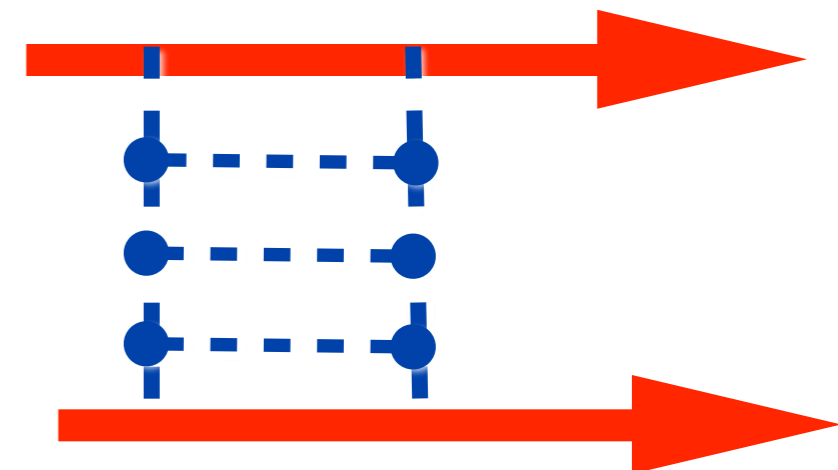
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



AdS/QCD:

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

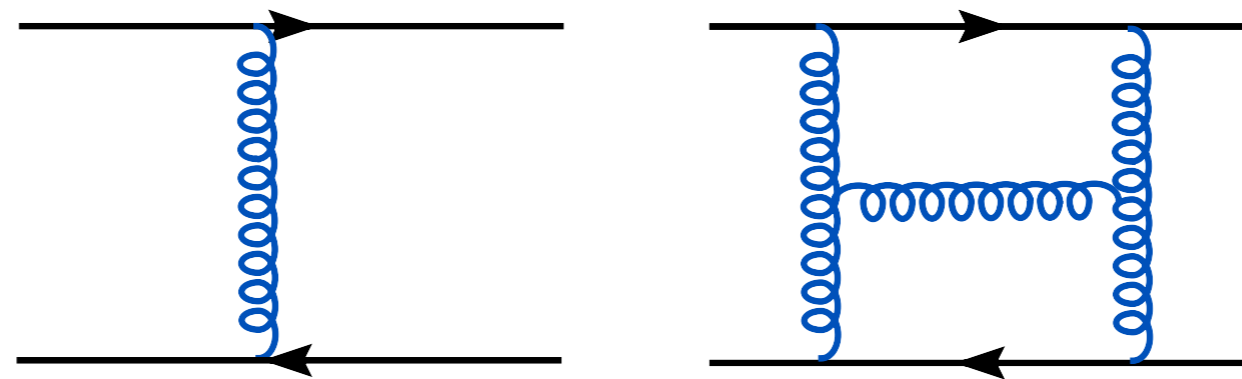
U is the exact QCD potential
Conjecture: 'H'-diagrams generate U?



Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \left[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2 + (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \right]$$

Smirnov, Smirnov, Steinhauser, 2010

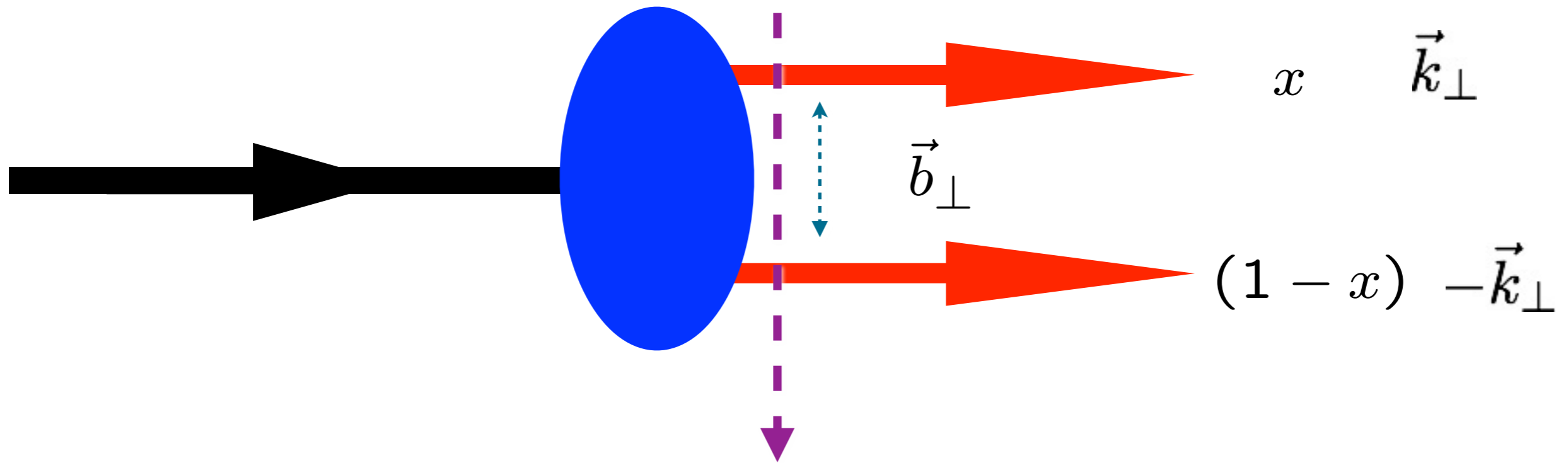


$\log \kappa^2 \zeta^2$

Summation of H graphs: confining potential

*Confinement eliminates IR divergences
Self-consistent mass scale κ*

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

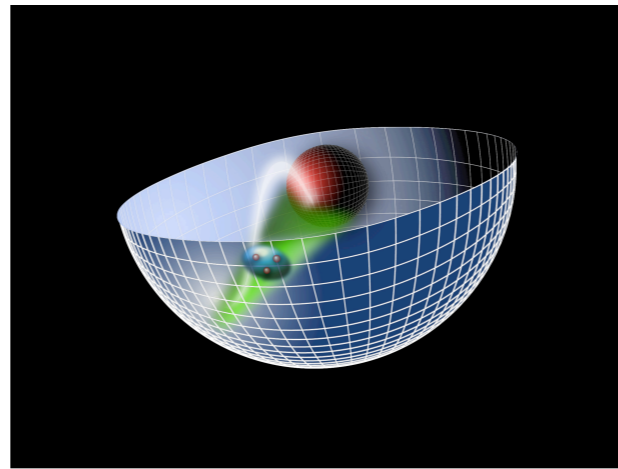
Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}_{q+\bar{q}}^2$$

$$\int dk^- \Psi_{BS}(P, k) \rightarrow \psi_{LF}(x, \vec{k}_\perp)$$

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

Confinement scale:

$$1/\kappa \simeq 1/3 \text{ fm}$$

***Unique
Confinement Potential!***

*Preserves Conformal Symmetry
of the action*

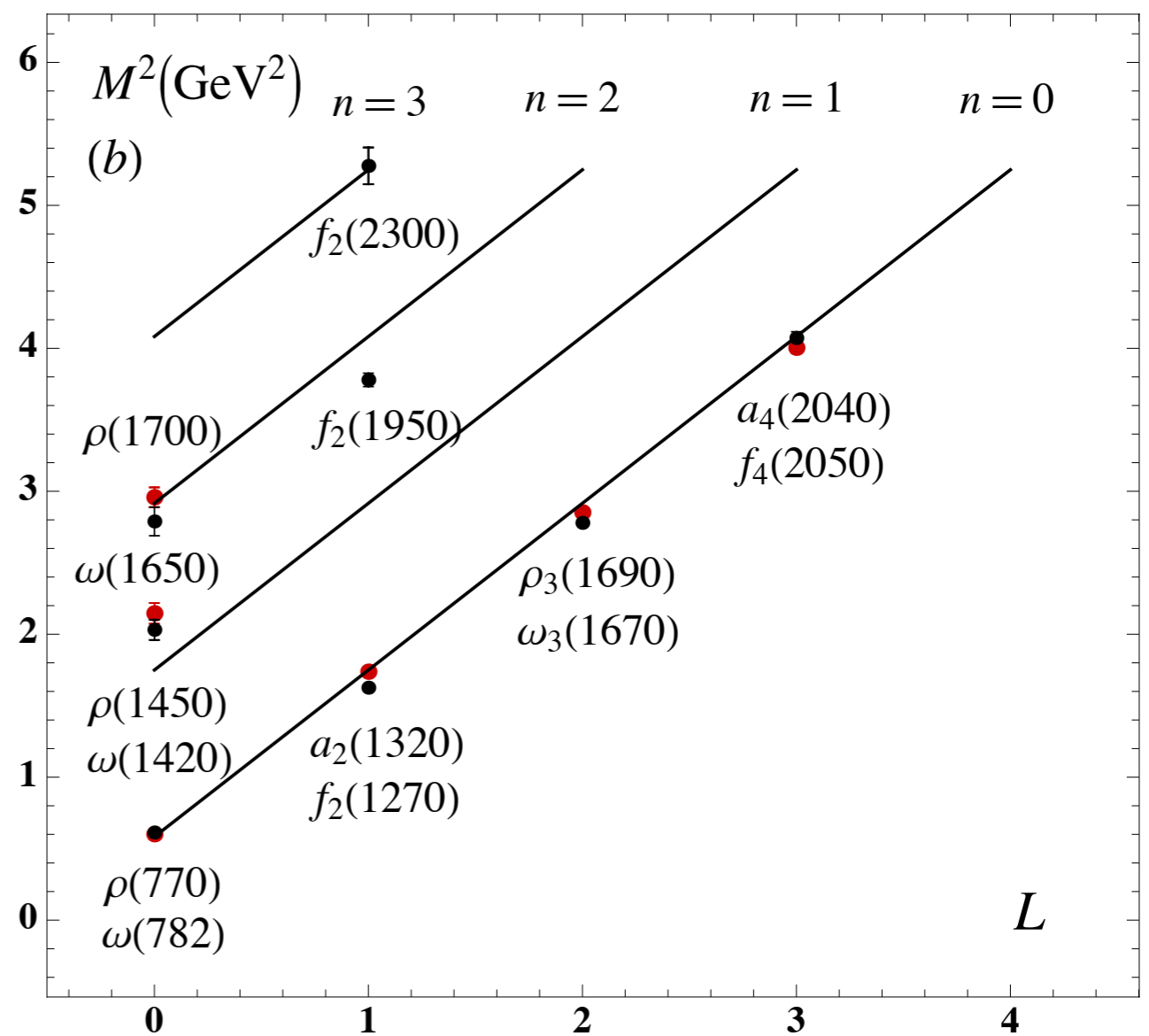
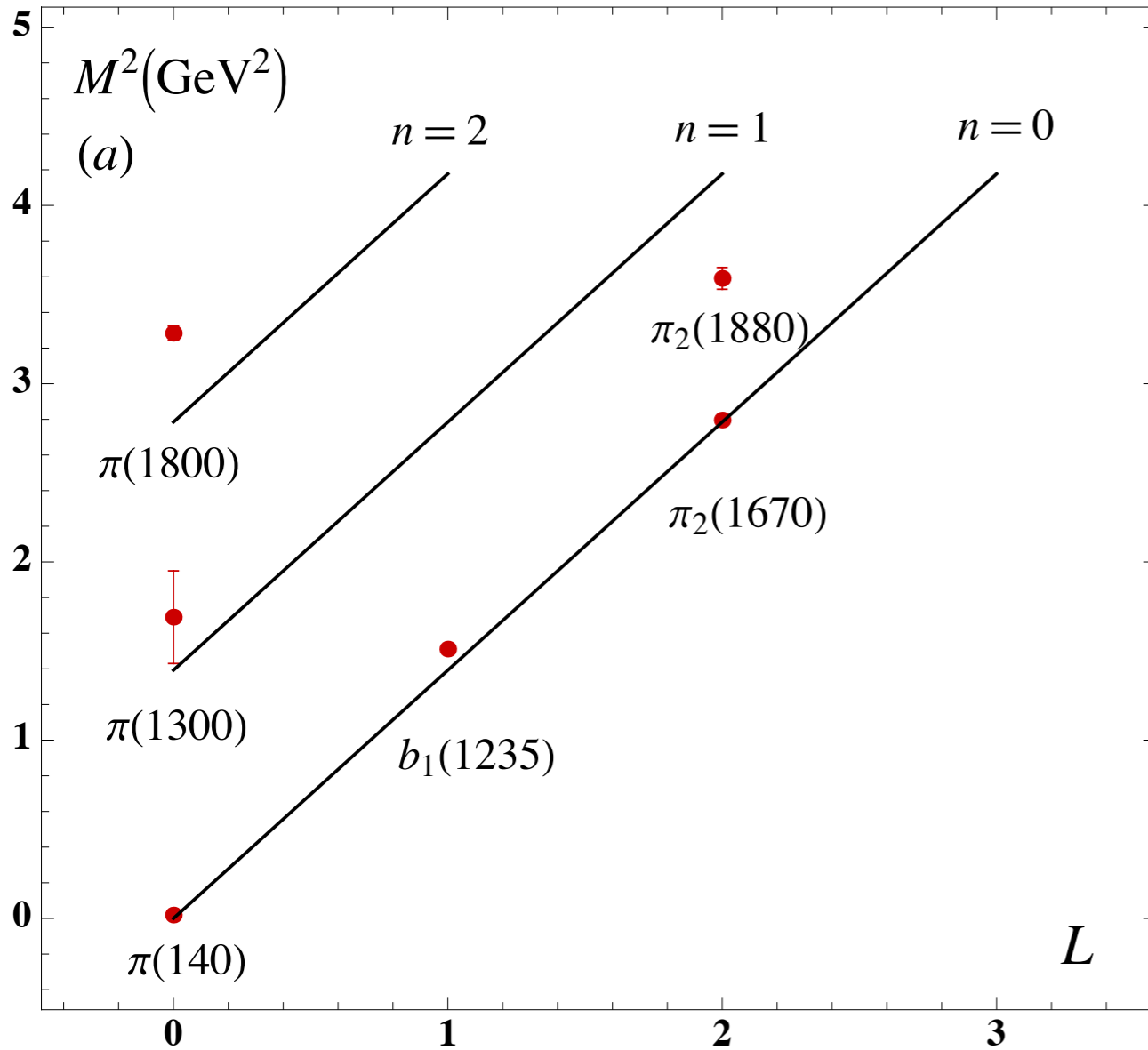
● **de Alfaro, Fubini, Furlan:**

● **Fubini, Rabinovici:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

$$m_u = m_d = 0$$

Preview



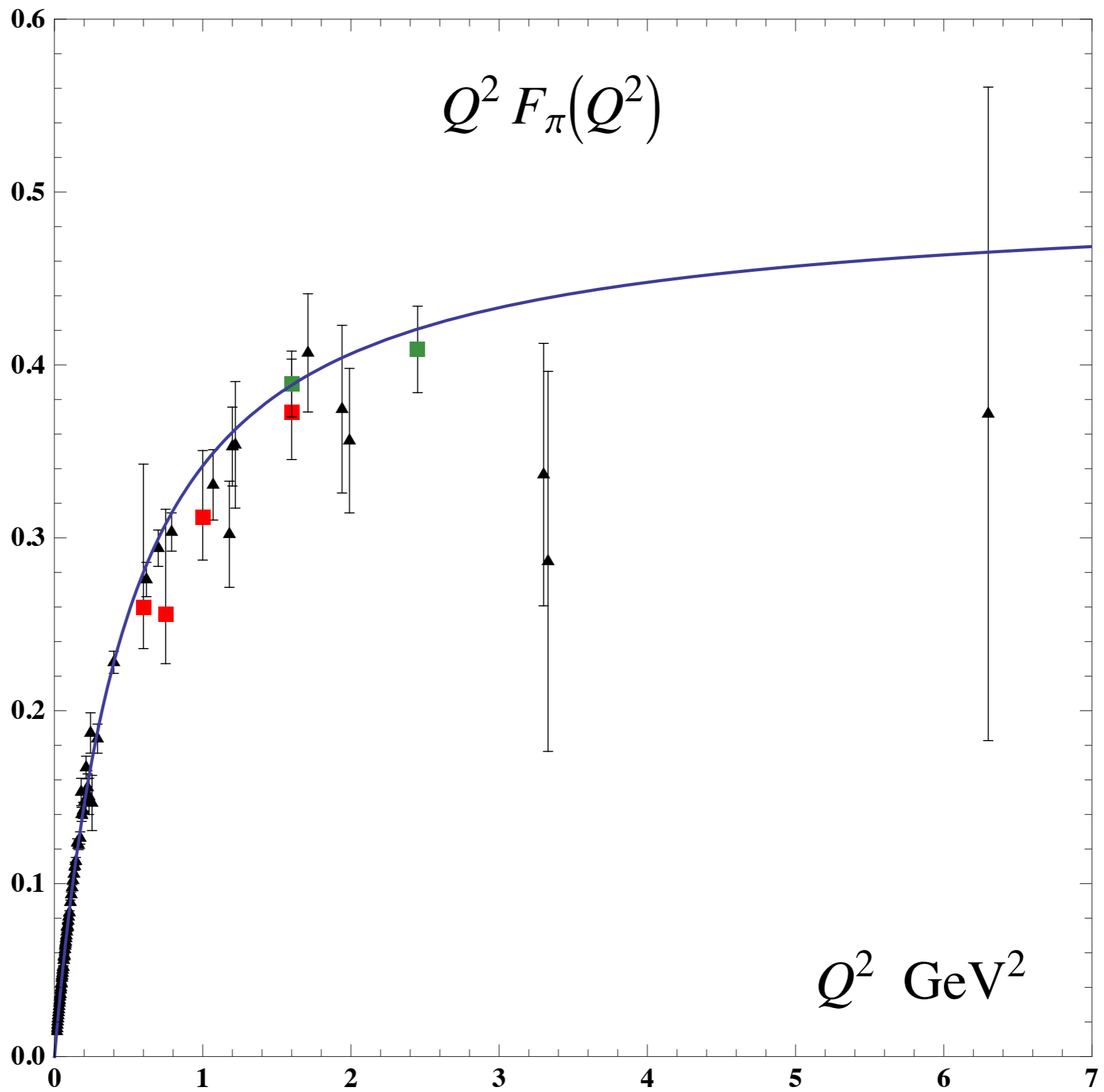
$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

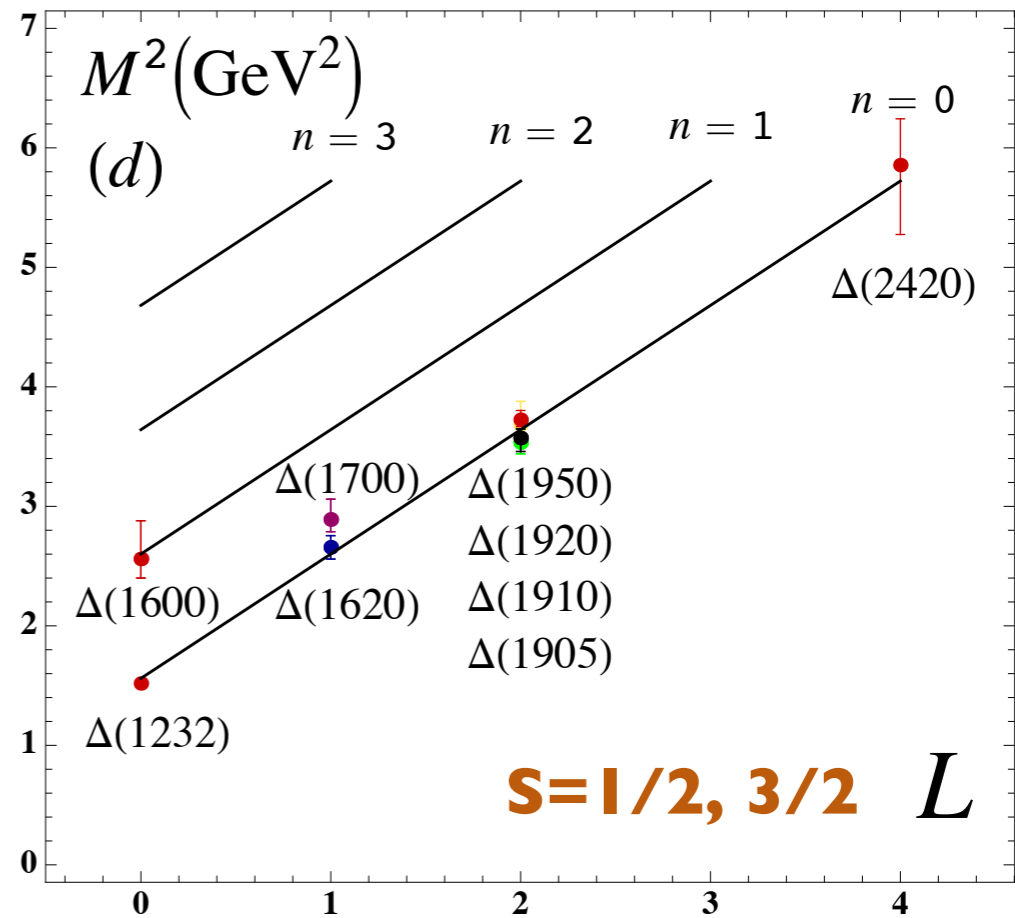
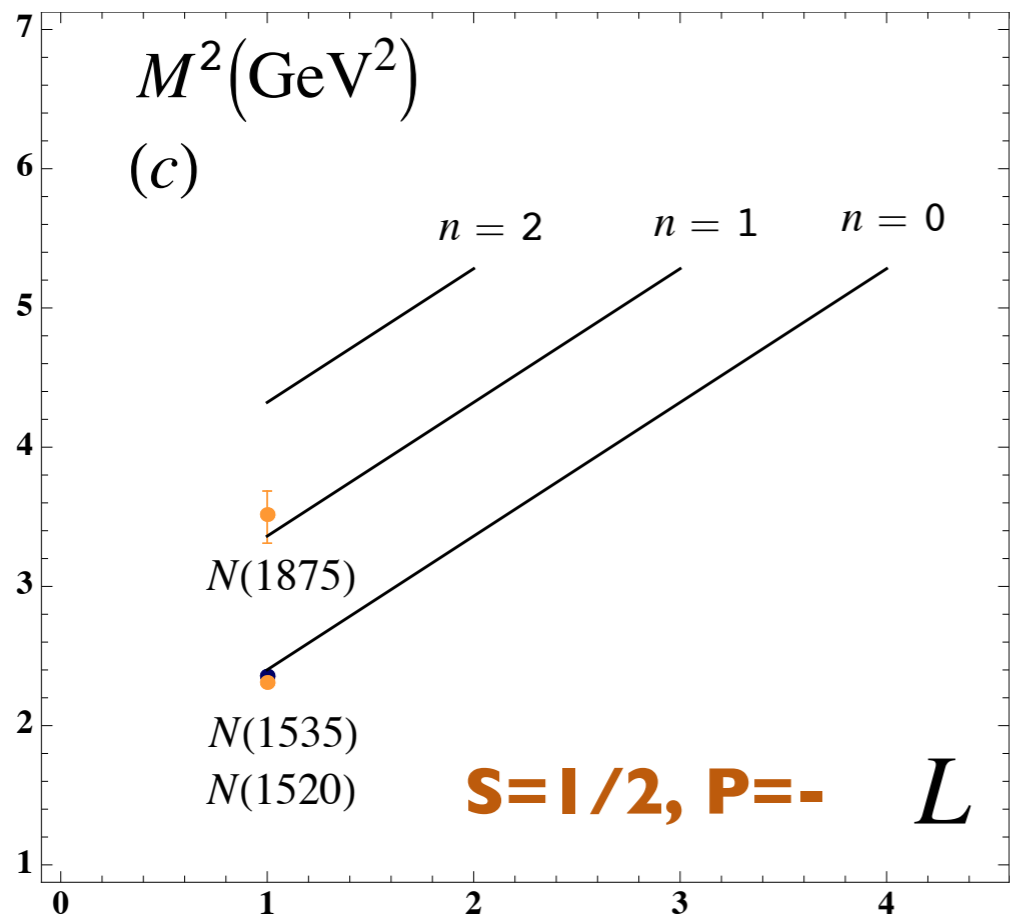
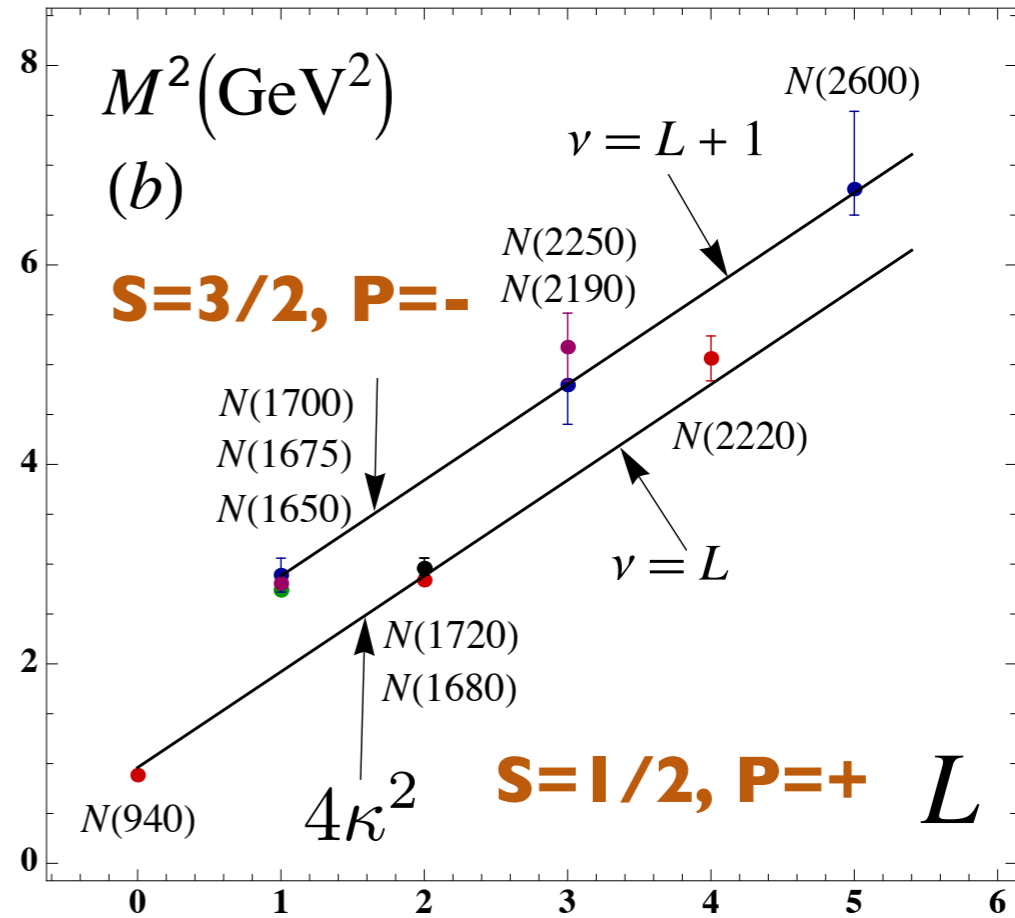
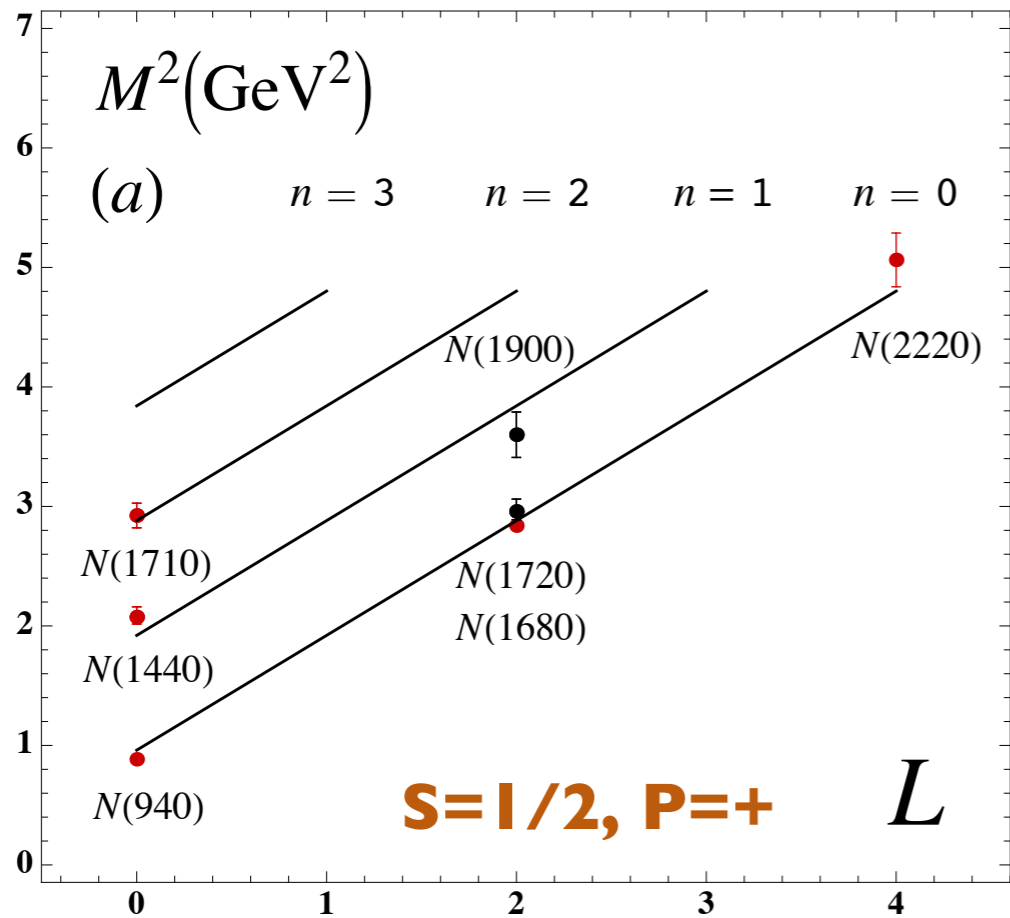
Slovenia
July 2015



*New Perspectives for Hadron Physics
and the Cosmological Constant*

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY

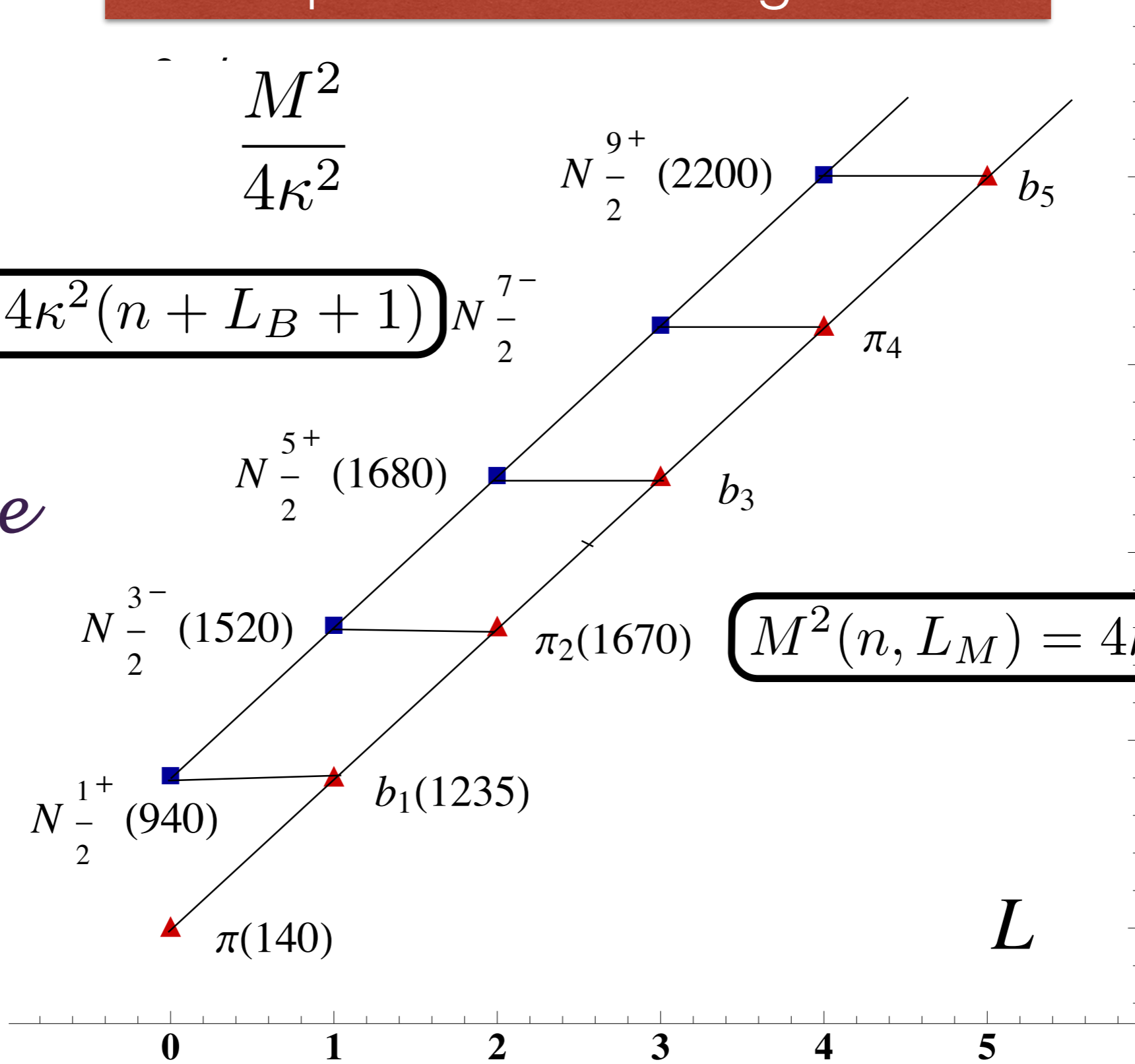




Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

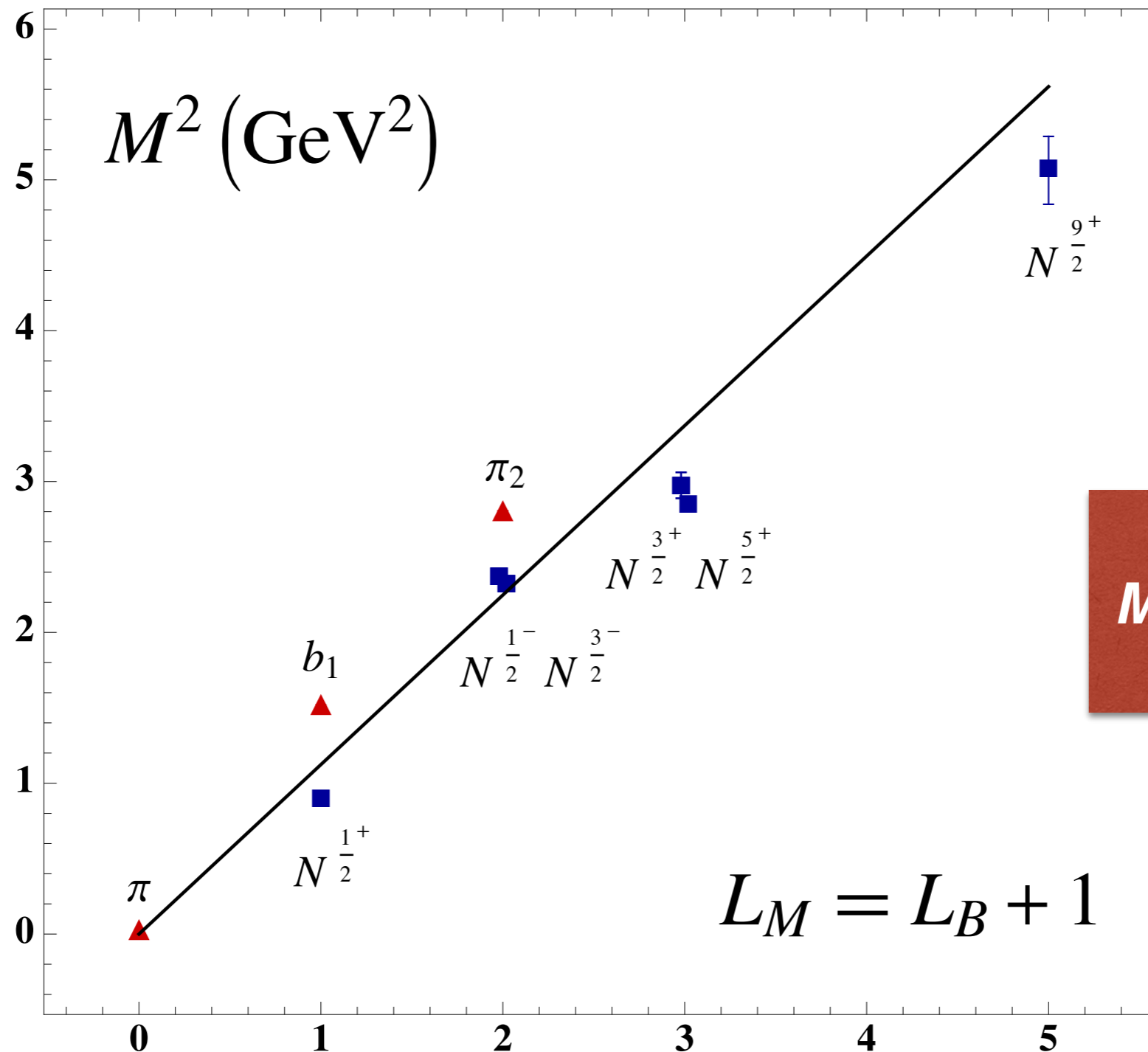
Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

**Meson-Baryon
Mass Degeneracy
for $L_M=L_B+1$**

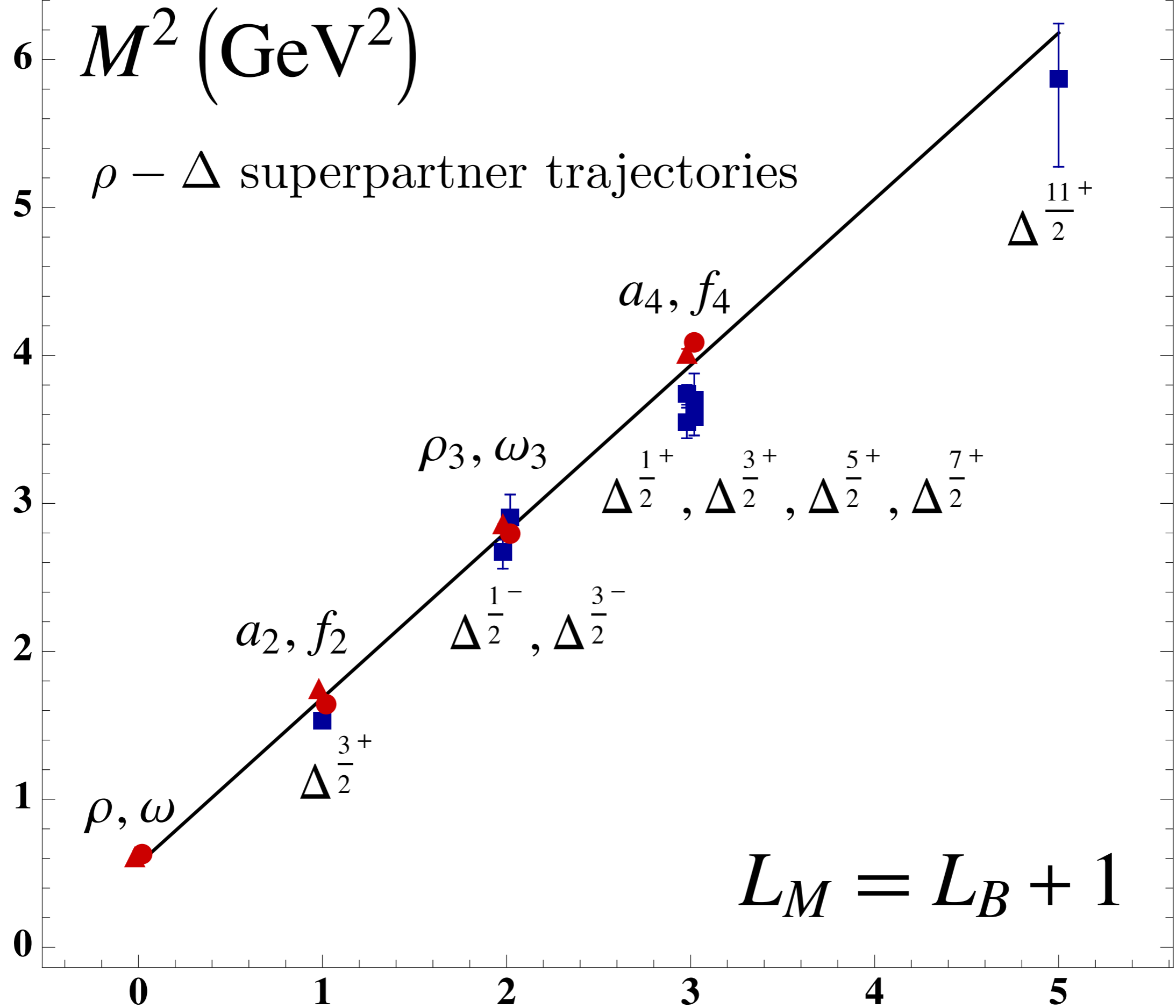
Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



$S=0, I=I$ Meson is superpartner of $S=1/2, I=I$ Baryon

M^2 (GeV²)

$\rho - \Delta$ superpartner trajectories



Some Features of AdS/QCD

- **Regge spectroscopy—same slope in n, L for mesons,**
- **Chiral features for $m_q=0$: $m_\pi=0$, chiral-invariant proton**
- **Hadronic LFWFs**
- **Counting Rules**
- **Connection between hadron masses and $\Lambda_{\overline{MS}}$**

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$



$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa_1$$

Deur, de Tèramond, sjb

All-Scale QCD Coupling

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$

$$e^{-\frac{Q^2}{4\kappa^2}}$$

Expt:
 $\Lambda_{\overline{MS}} = 0.341 \pm 0.024 \text{ GeV}$

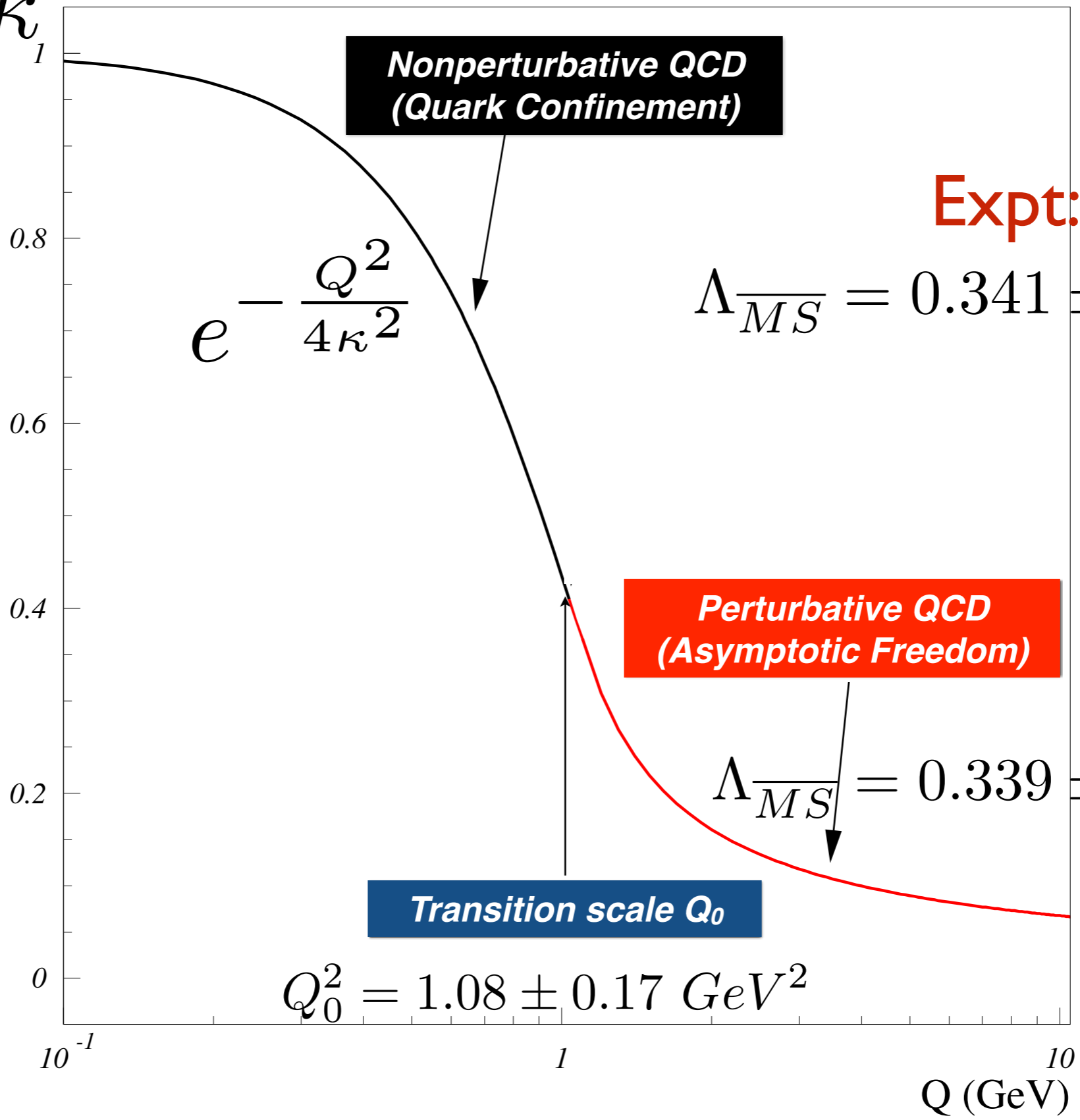
**Perturbative QCD
(Asymptotic Freedom)**

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016 \text{ GeV}$$

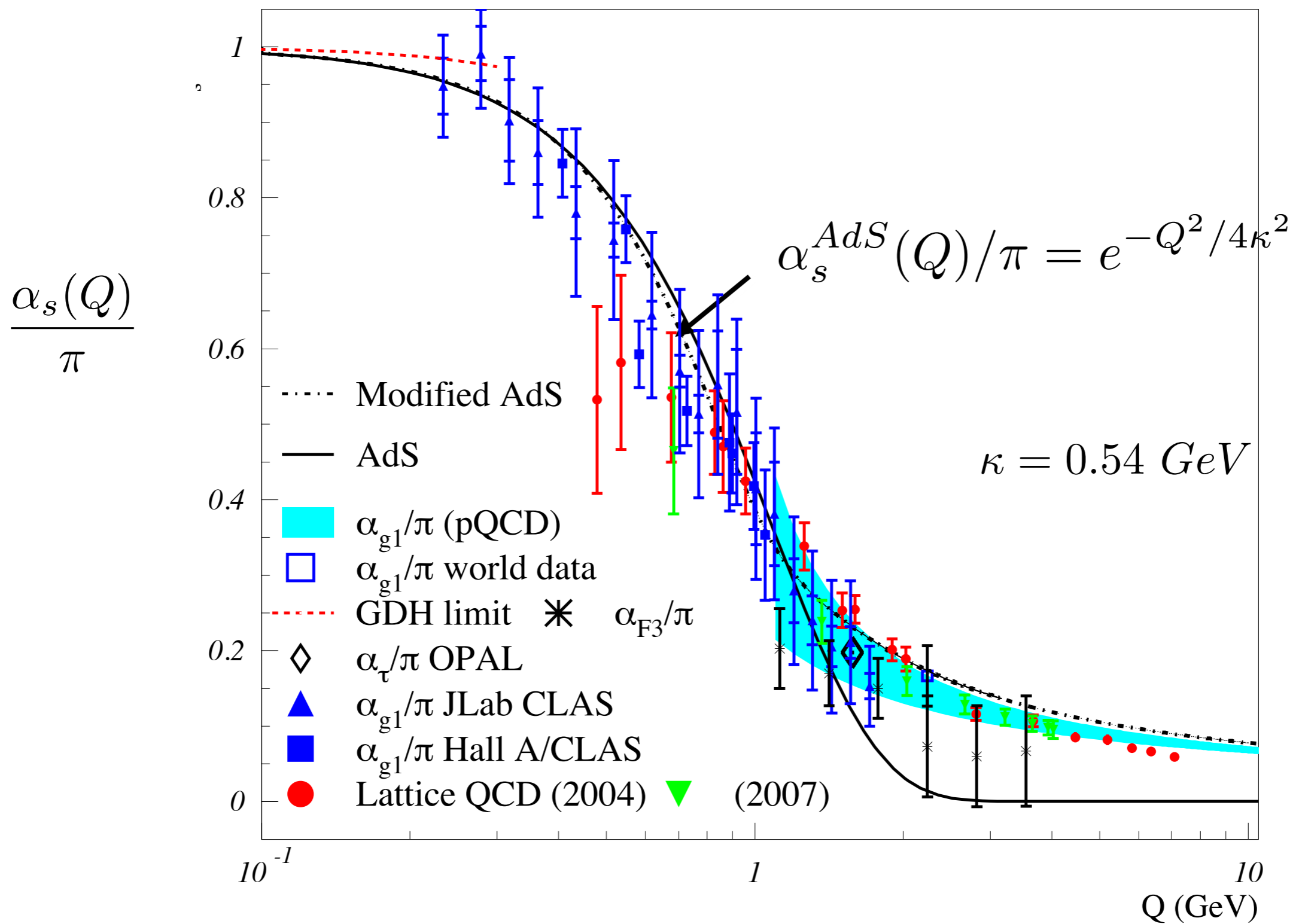
Transition scale Q_0

$$Q_0^2 = 1.08 \pm 0.17 \text{ GeV}^2$$

$$\lambda \equiv \kappa^2$$



Analytic, defined at all scales, IR Fixed Point



$$e^\varphi = e^{+\kappa^2 z^2}$$

Deur, de Tèramond, sjb

● **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**

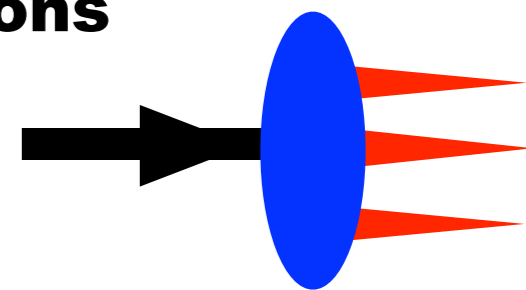
● **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**

● **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**

● **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, modulo 'lensing' from ISIs, FSIs**

● **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**

● **Hadron Physics without LFWFs is like Biology without DNA!**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

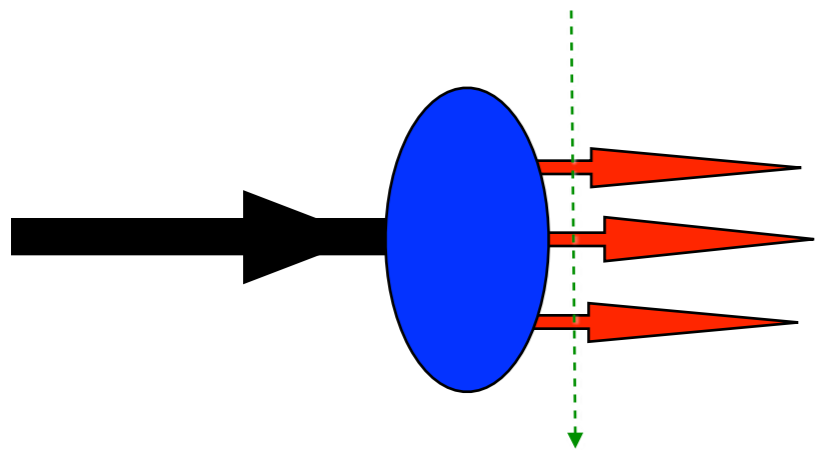


Advantages of the Dirac's Front Form for Hadron Physics

- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent -- no boosts!**
- **No dependence on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no condensates!**
- **Profound implications for Cosmological Constant**



Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

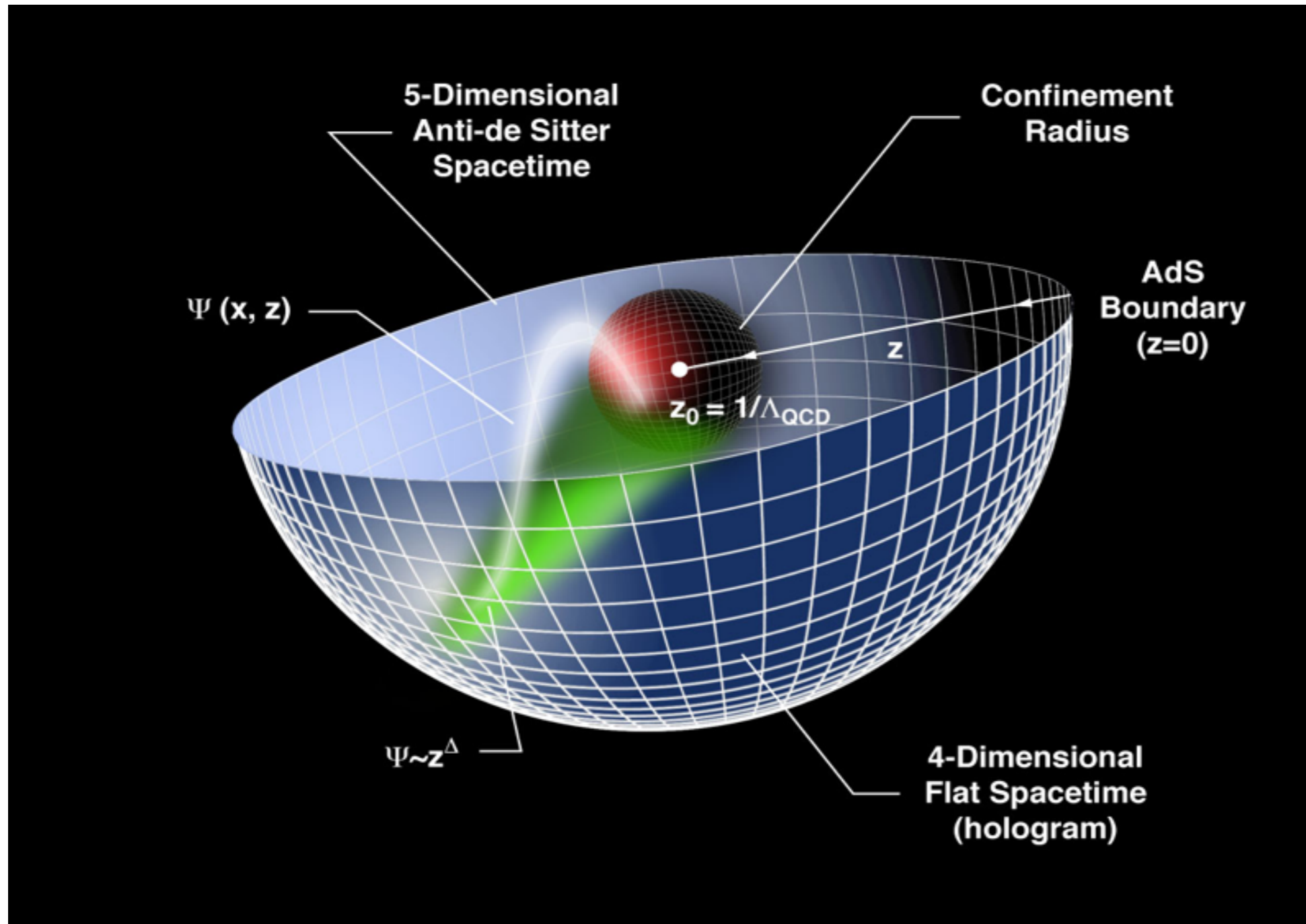
$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space



Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

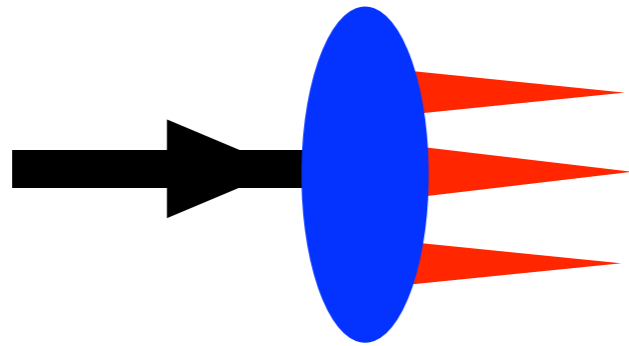
in collaboration with Guy de Teramond and H. Guenter Dosch

Light-Front Holography and Non-Perturbative QCD

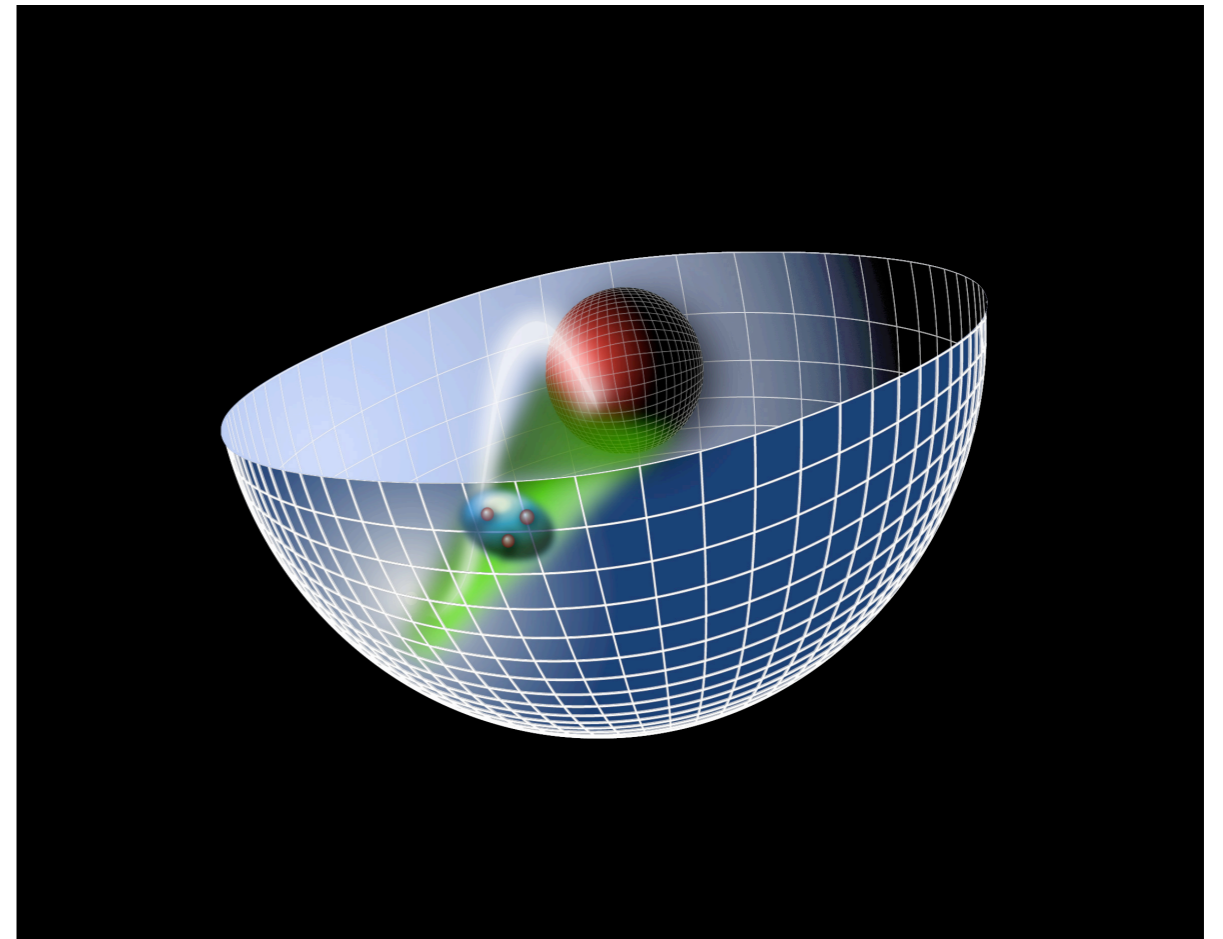
Goal:

**Use AdS/QCD duality to construct
a first approximation to QCD**

*Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$




in collaboration with Guy de Teramond and H. Guenter Dosch

AdS/CFT

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

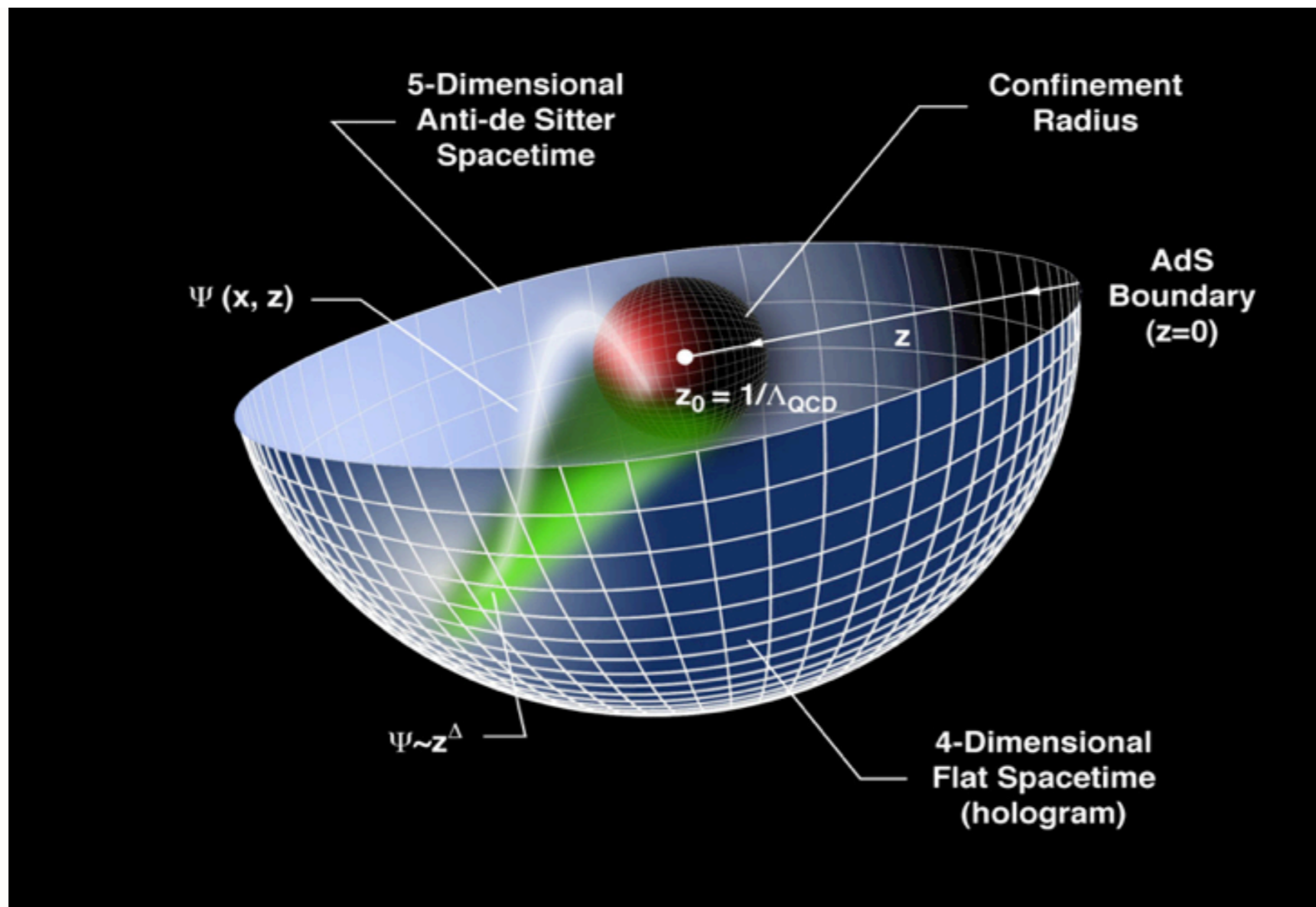
$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.



8-2007
8685A14

Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

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July 2015



*New Perspectives for Hadron Physics
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Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS₅ as template for conformal theory**



Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

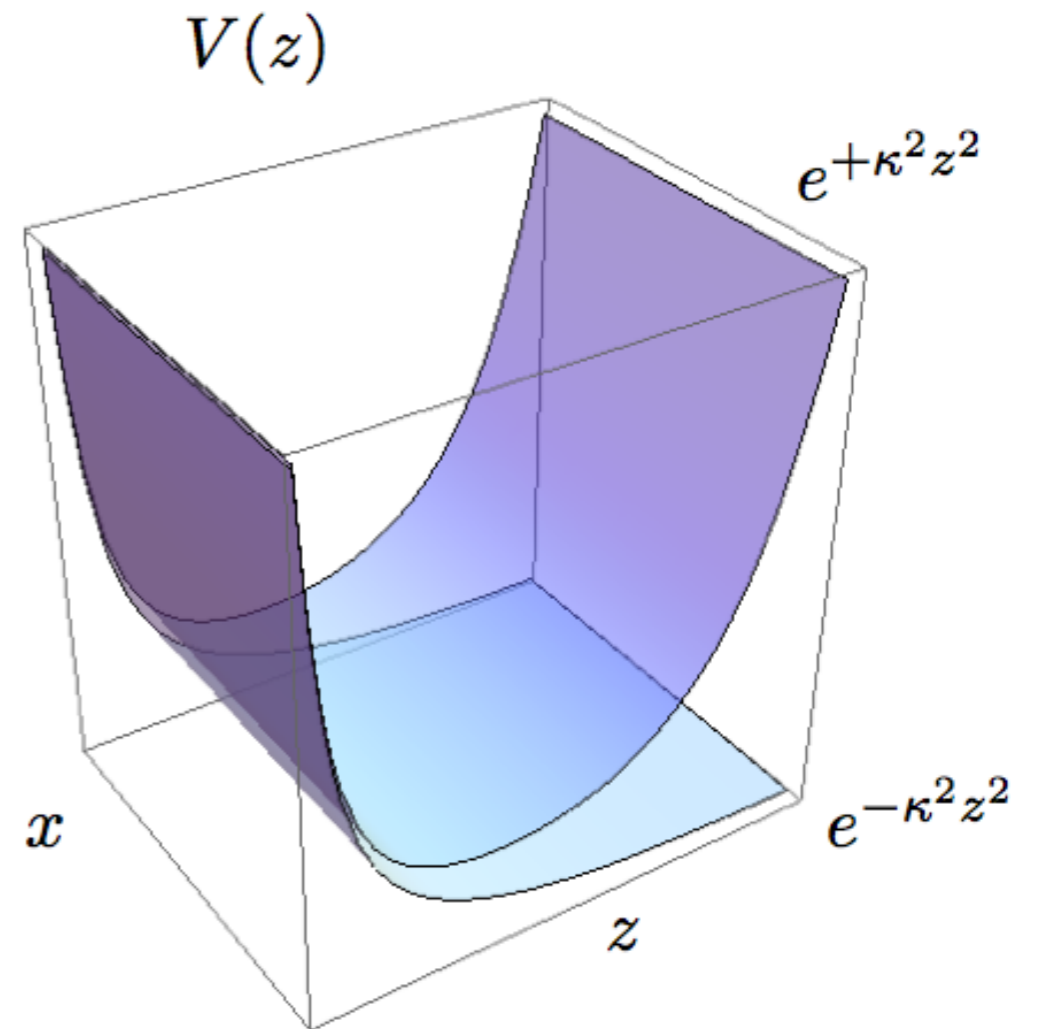
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

where $\varphi(z) \rightarrow 0$ at small z for geometries which are asymptotically AdS₅

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$.

- Action for massive scalar modes on AdS_{d+1} :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ -coordinates, $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \rightarrow z^\Delta$ as $z \rightarrow 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

• Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

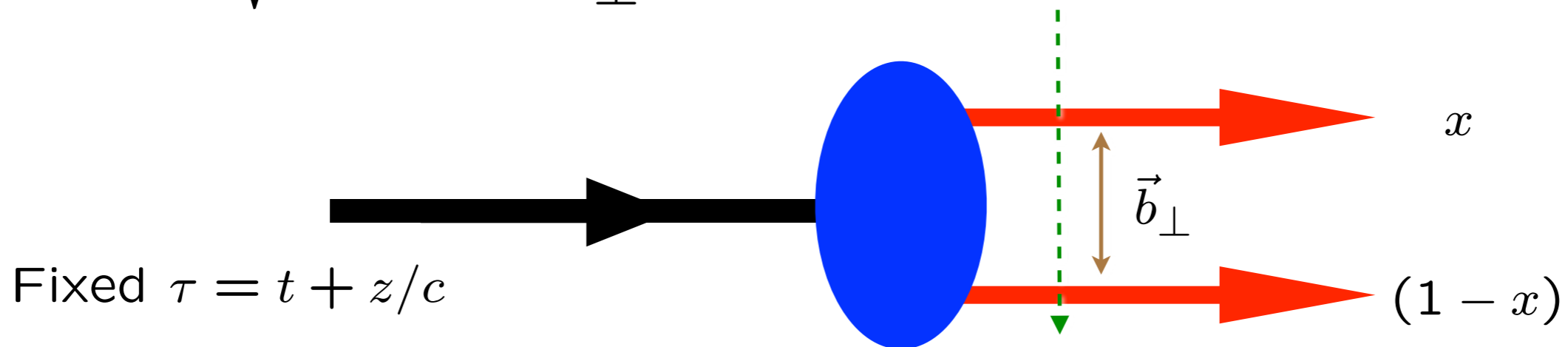
Derived from variation of Action for Dilaton-Modified AdS₅

Identical to Light-Front Bound State Equation!

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

$LF(3+1) \longleftrightarrow AdS_5$

Light-Front Holographic Dictionary

 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$
 $\zeta = \sqrt{x(1-x)b_\perp^2} \longleftrightarrow z$


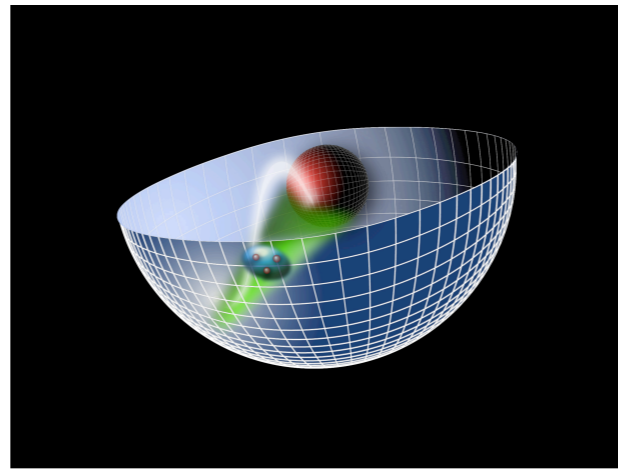
$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

Confinement scale:

$$1/\kappa \simeq 1/3 \text{ fm}$$

***Unique
Confinement Potential!***

*Preserves Conformal Symmetry
of the action*

● **de Alfaro, Fubini, Furlan:**

● **Fubini, Rabinovici:**

***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!***

General-Spin Hadrons

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$



with $(\mu R)^2 = -(2 - J)^2 + L^2$

Meson Spectrum in Soft Wall Model

Pion: Negative term for $J=0$ cancels positive terms from LFKE and potential



- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

- $J = L + S, I = 1$ meson families

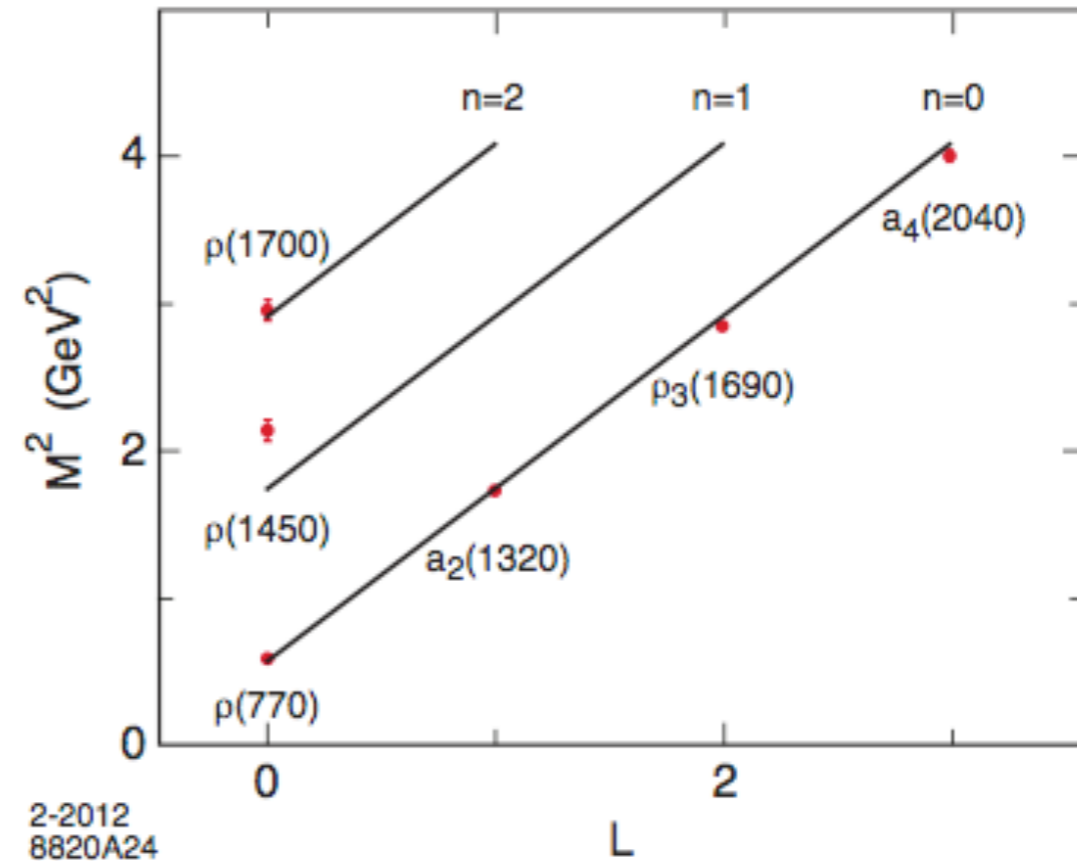
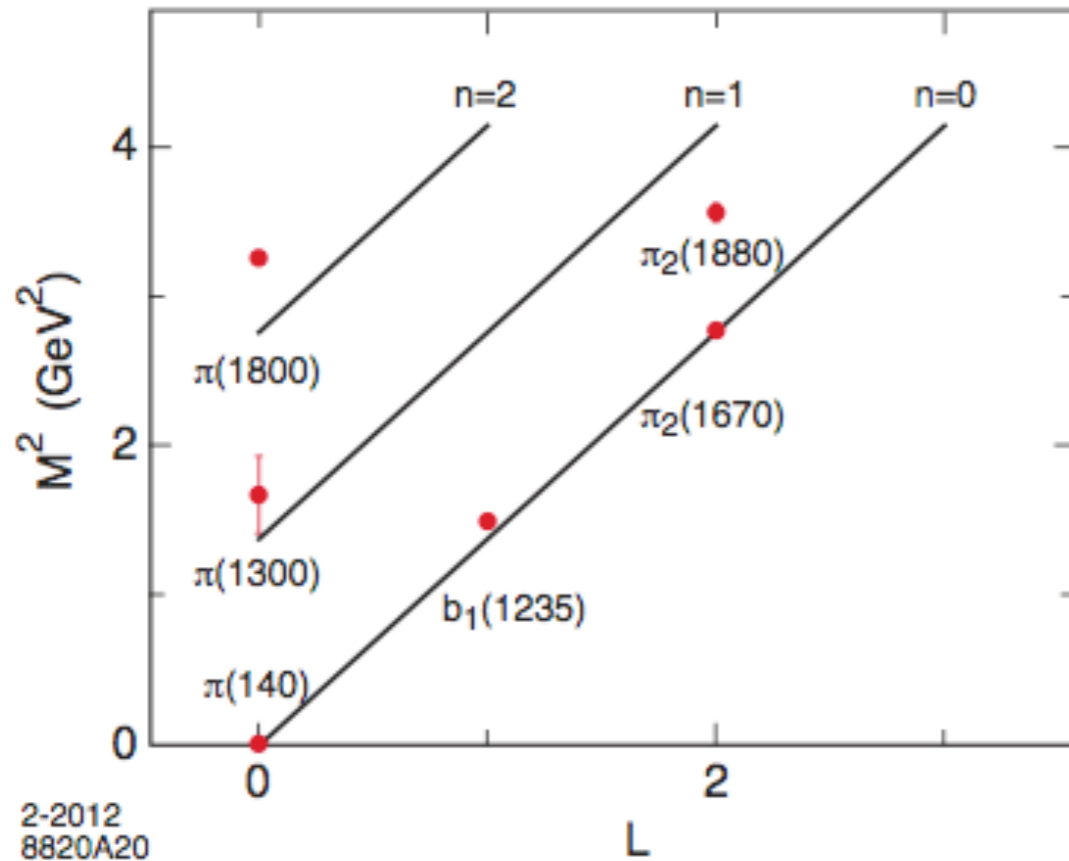
$$\mathcal{M}_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$$

$$\begin{aligned} 4\kappa^2 &\text{ for } \Delta n = 1 \\ 4\kappa^2 &\text{ for } \Delta L = 1 \\ 2\kappa^2 &\text{ for } \Delta S = 1 \end{aligned}$$

$$m_q = 0$$

Massless pion in Chiral Limit!

Same slope in n and L !



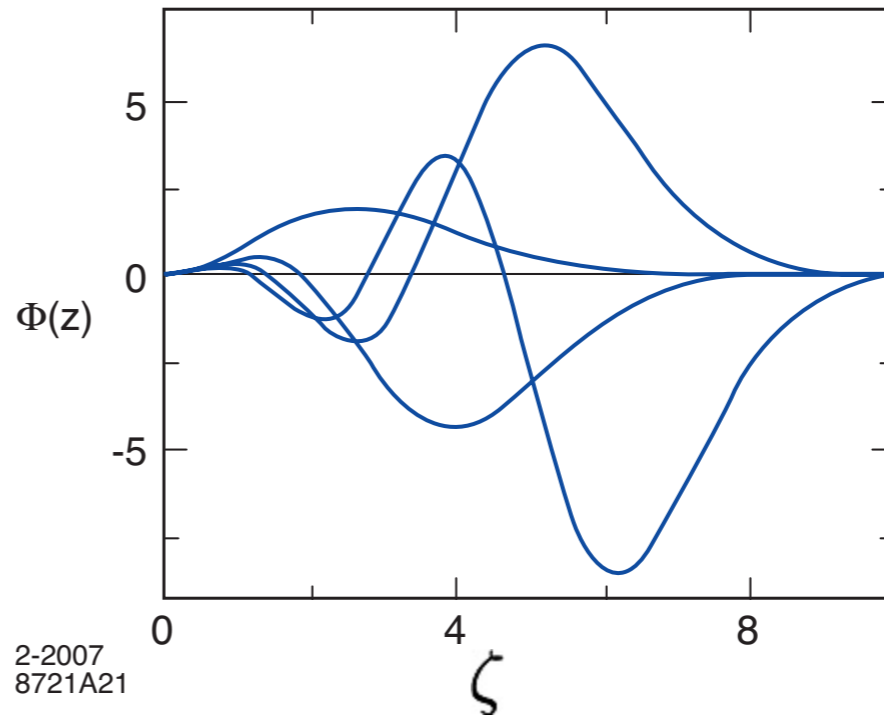
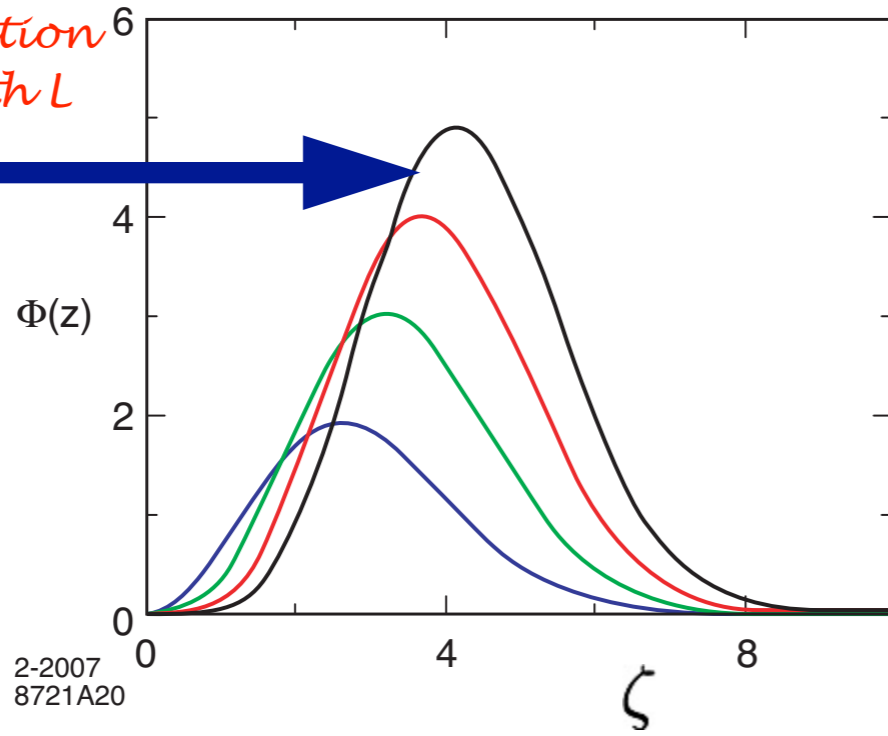
$I=1$ orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ -meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a -states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

Quark separation increases with L

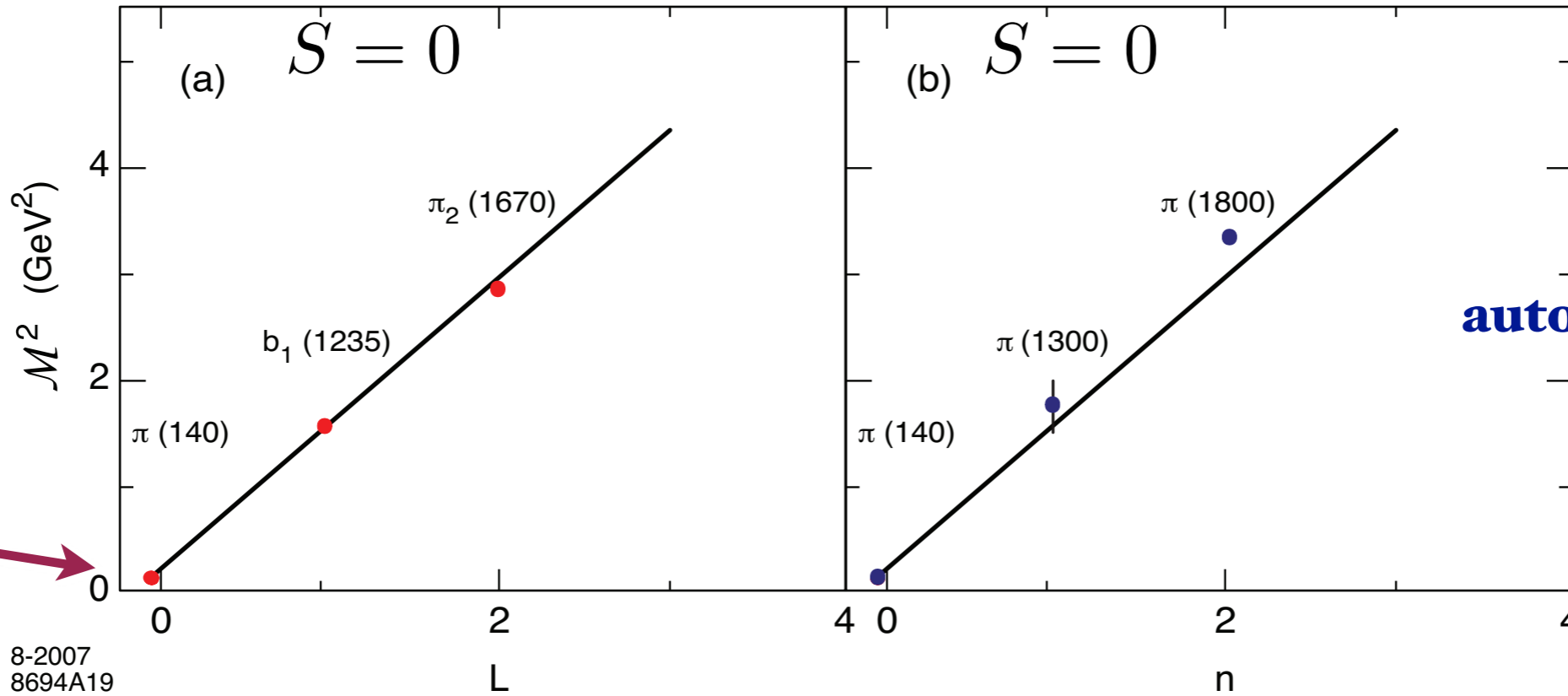


2-2007
8721A20

2-2007
8721A21

Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .
Same slope in n and L !

Soft Wall Model



8-2007
8694A19

Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

- Results easily extended to light quarks masses (Ex: K -mesons)
- First order perturbation in the quark masses

$$\Delta M^2 = \langle \psi | \sum_a m_a^2 / x_a | \psi \rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x, \zeta) \sim \sqrt{x(1-x)} e^{-\frac{1}{2\lambda} \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right)} e^{-\frac{1}{2} \lambda \zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA
[J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

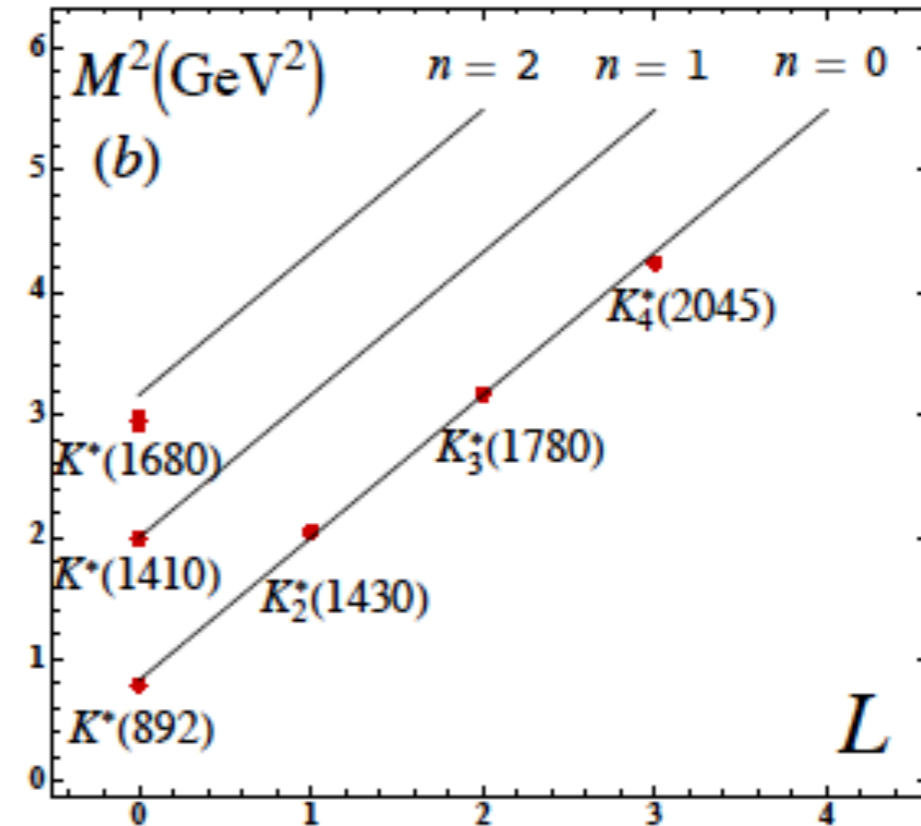
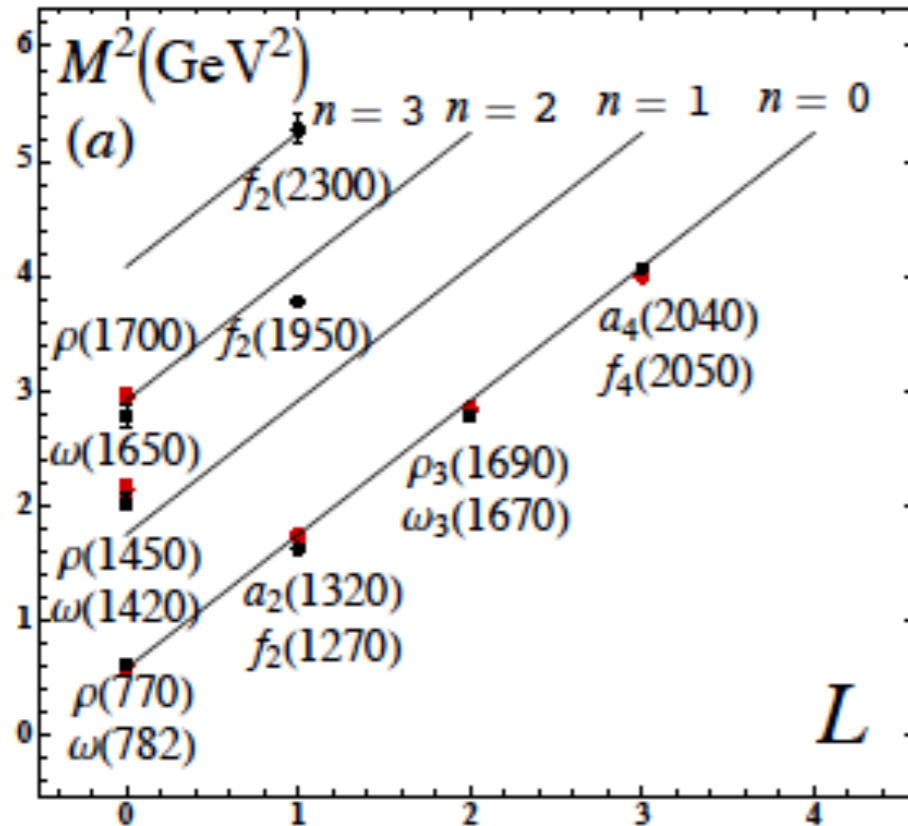
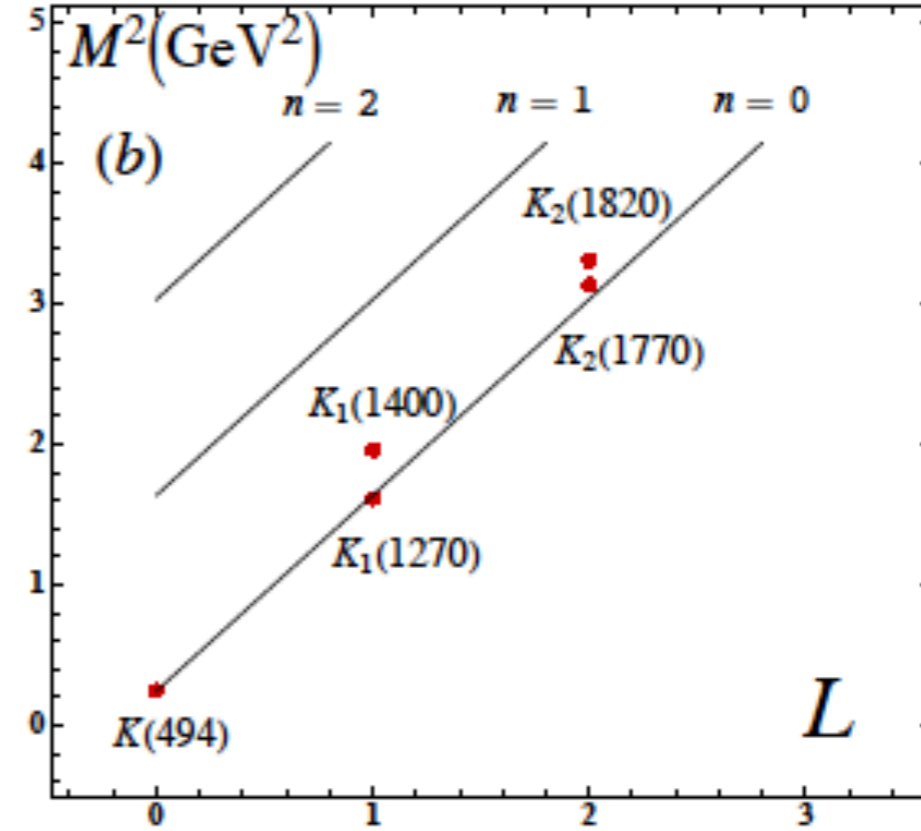
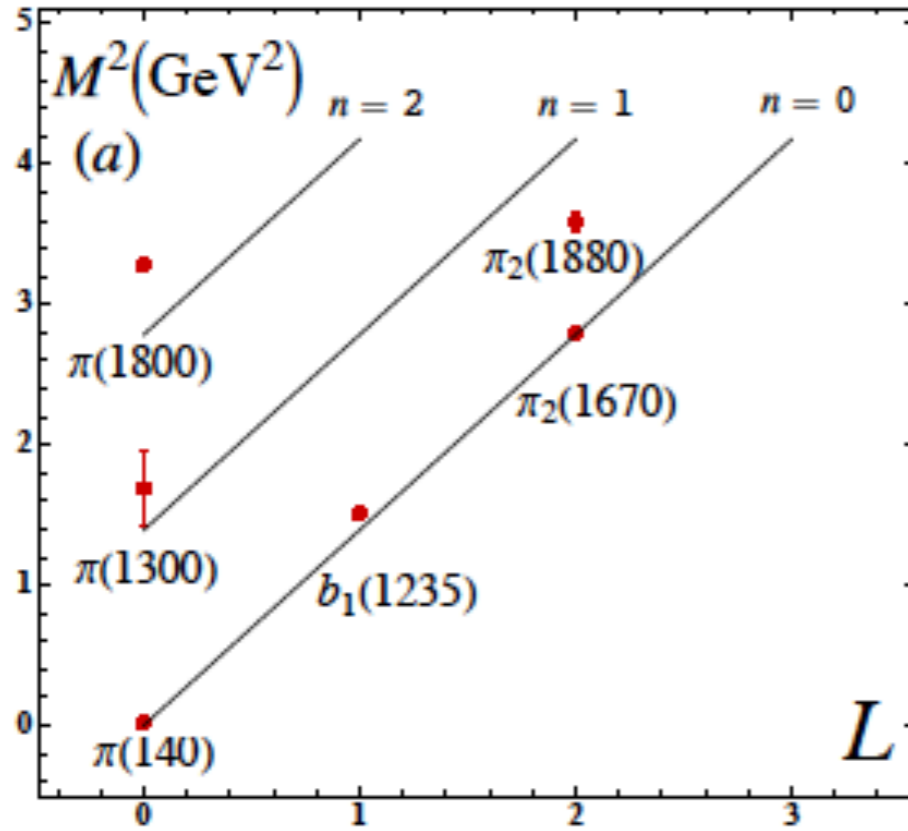
- For the K^*

$$M_{n,L,S}^2 = M_{K^\pm}^2 + 4\lambda \left(n + \frac{J+L}{2} \right)$$

- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



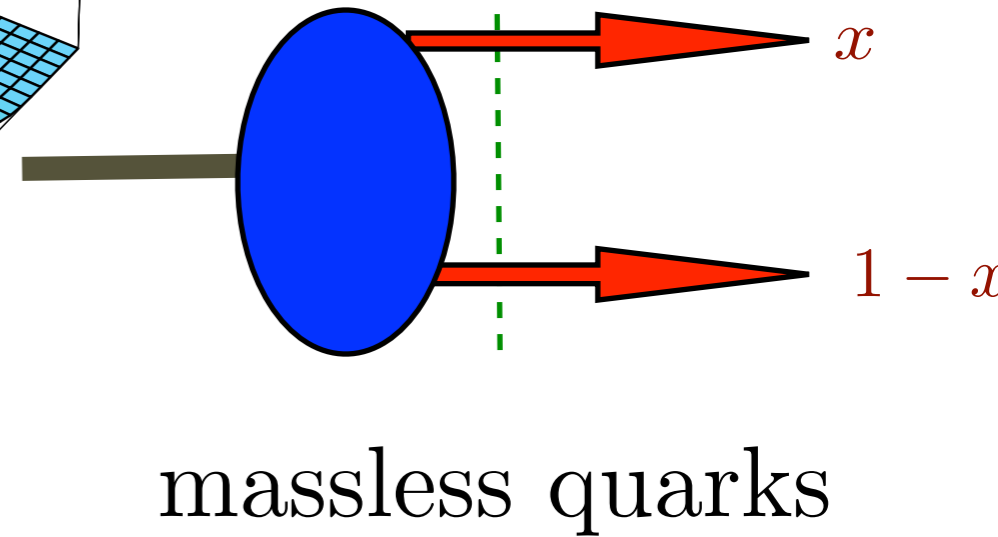
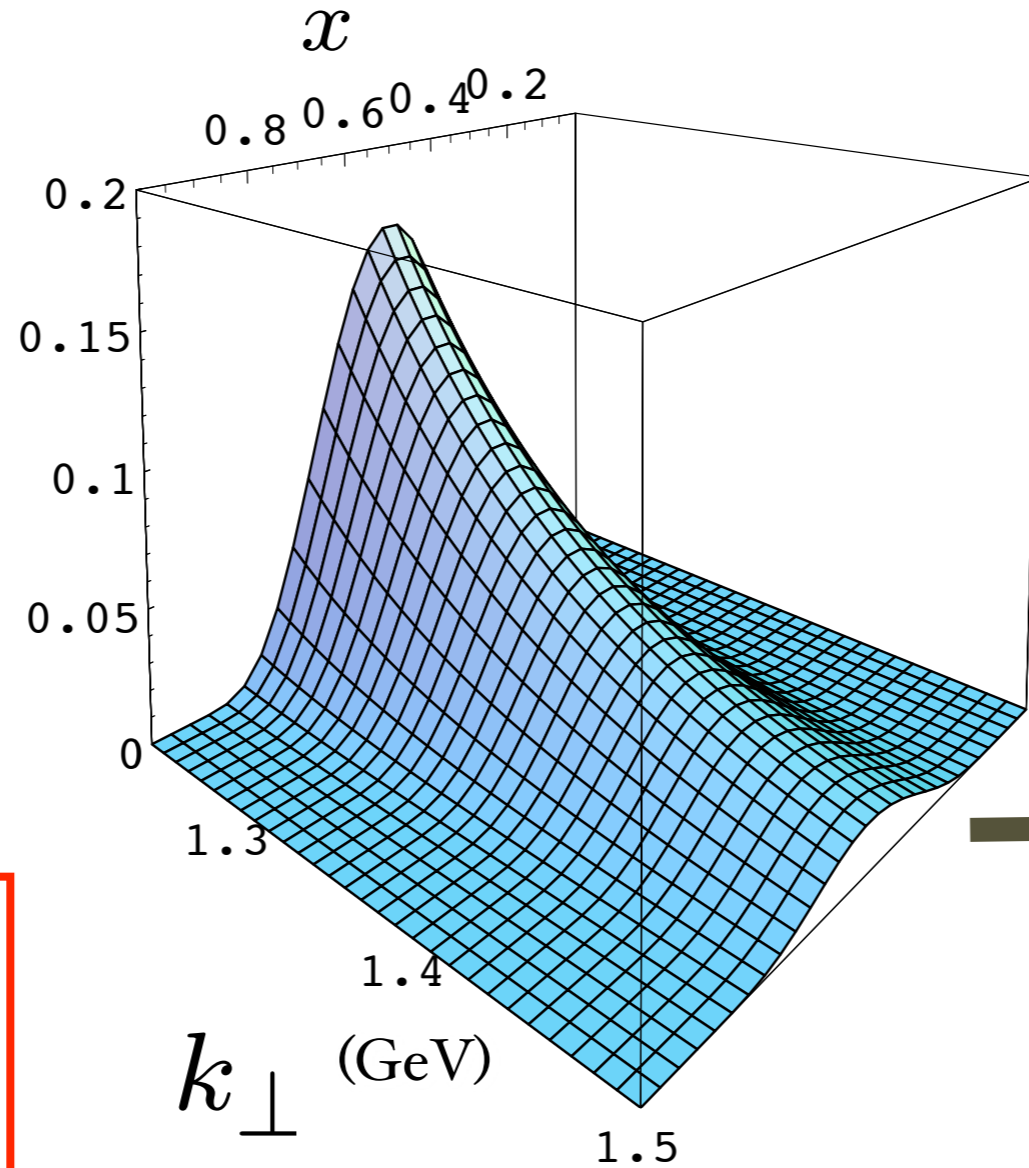
Prediction from AdS/QCD: Meson LFWF

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

de Teramond,
Cao, sjb

“Soft Wall”
model

$$\psi_M(x, k_{\perp}^2)$$



Note coupling

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_{\pi}(x) = \frac{4}{\sqrt{3}\pi} f_{\pi} \sqrt{x(1-x)}$$

$$f_{\pi} = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE!

Provides Connection of Confinement to Hadron Structure

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

J. R. Forshaw*

*Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester,
Oxford Road, Manchester M13 9PL, United Kingdom*

R. Sandapen†

Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada
(Received 5 April 2012; published 20 August 2012)

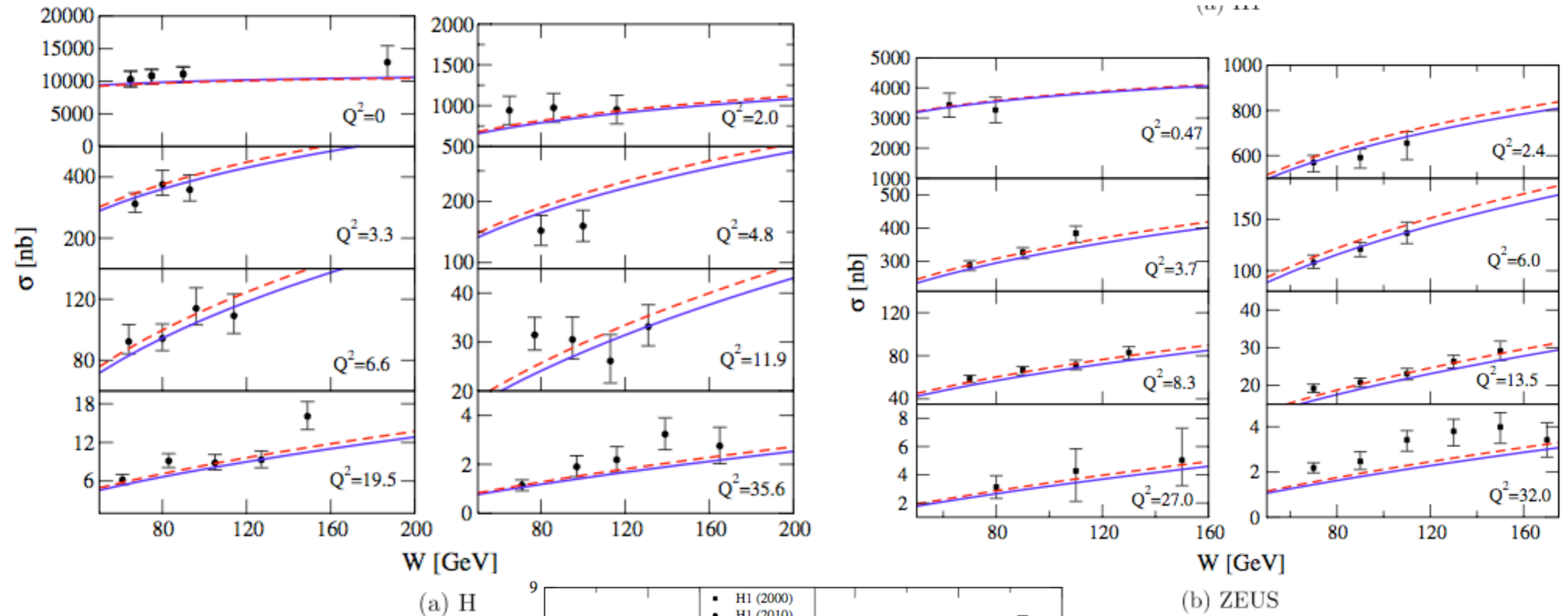
We show that anti-de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ -meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron-proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

**See also Ferreira
and Dosch**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

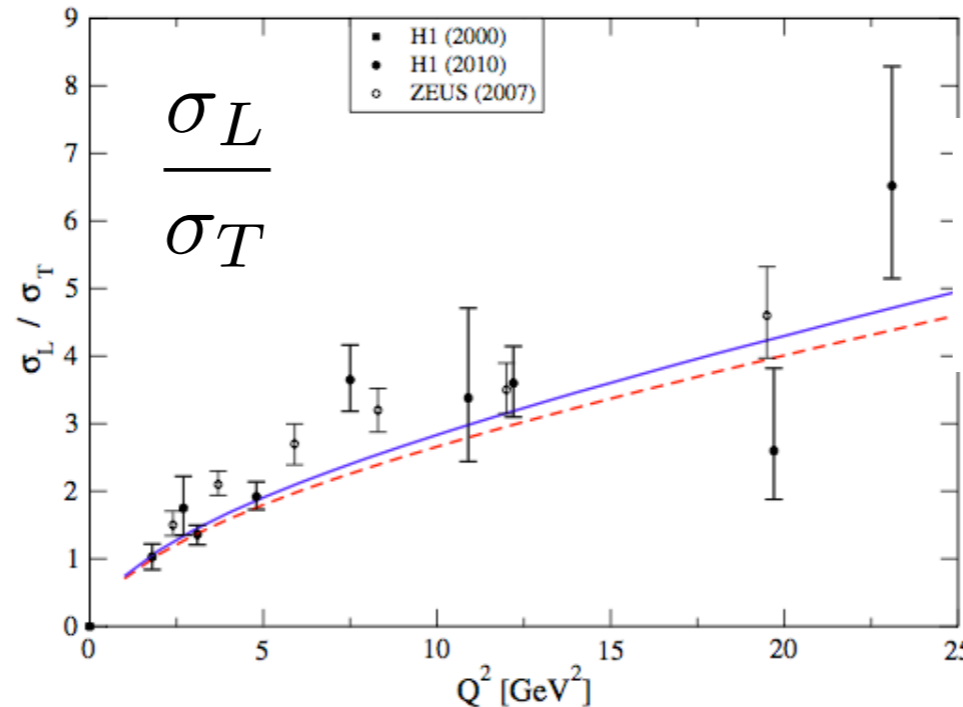


(a) H

(b) ZEUS

**J. R. Forshaw,
R. Sandapen**

$$\gamma^* p \rightarrow \rho^0 p'$$



$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right)$$

**See also Ferreira
and Dosch**

Uniqueness

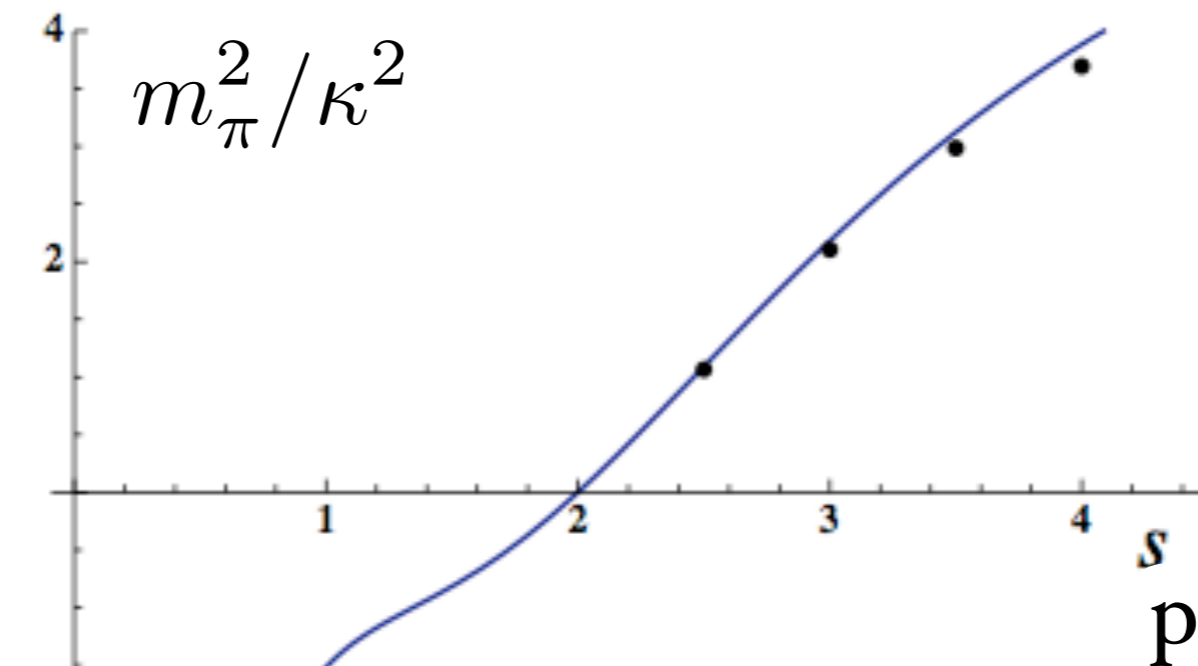
de Tèramond, Dosch, sjb

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \quad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L : same slope!**
- **Massless pion in chiral limit! No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini,
Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976)
569**

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff
 $p=2!$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Hadron Form Factors from AdS/QCD

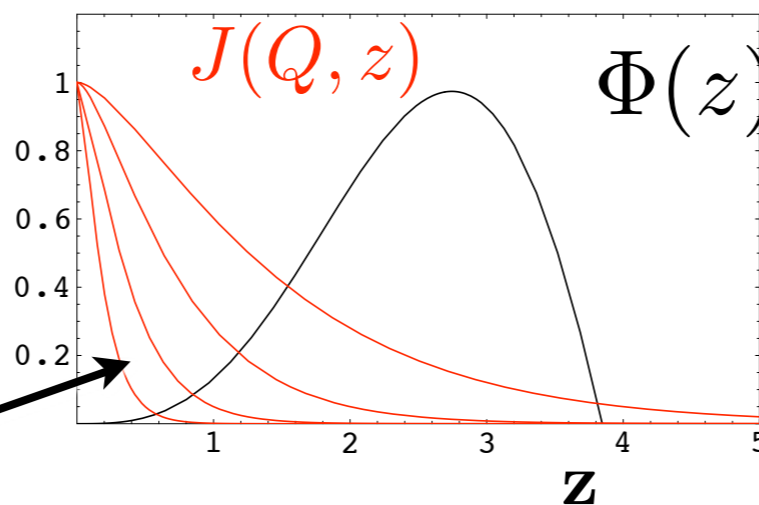
Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQ K_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$

high Q^2



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

$$\text{Twist } \tau = n + L$$

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$.

Holographic Mapping of AdS Modes to QCD LFWFs

*Drell-Yan-West: Form Factors are
Convolution of LFWFs*

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

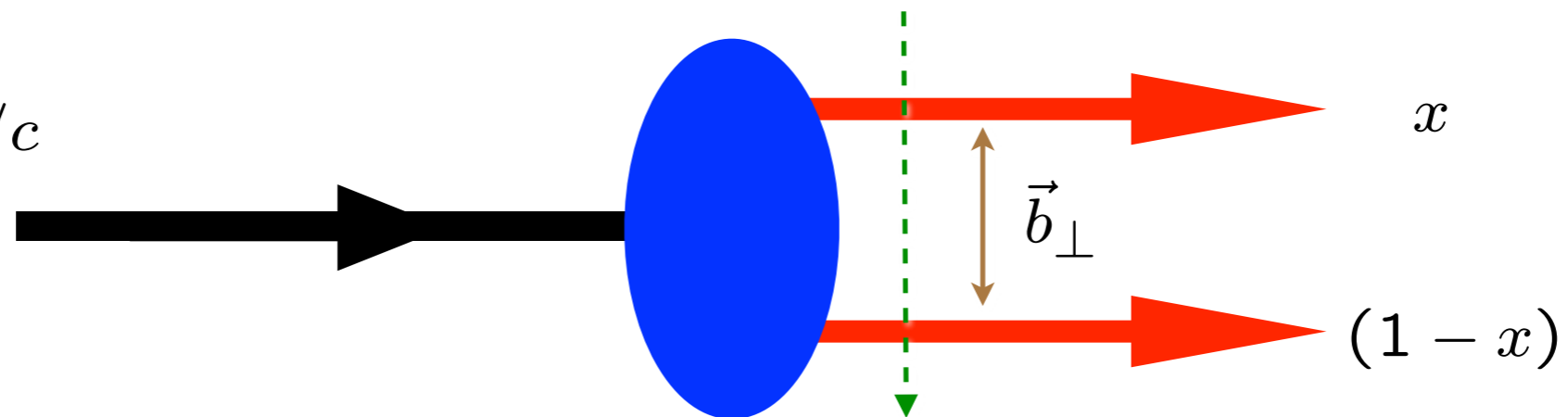
Identical to Polchinski-Strassler Convolution of AdS Amplitudes

$LF(3+1) \longleftrightarrow AdS_5$

$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$

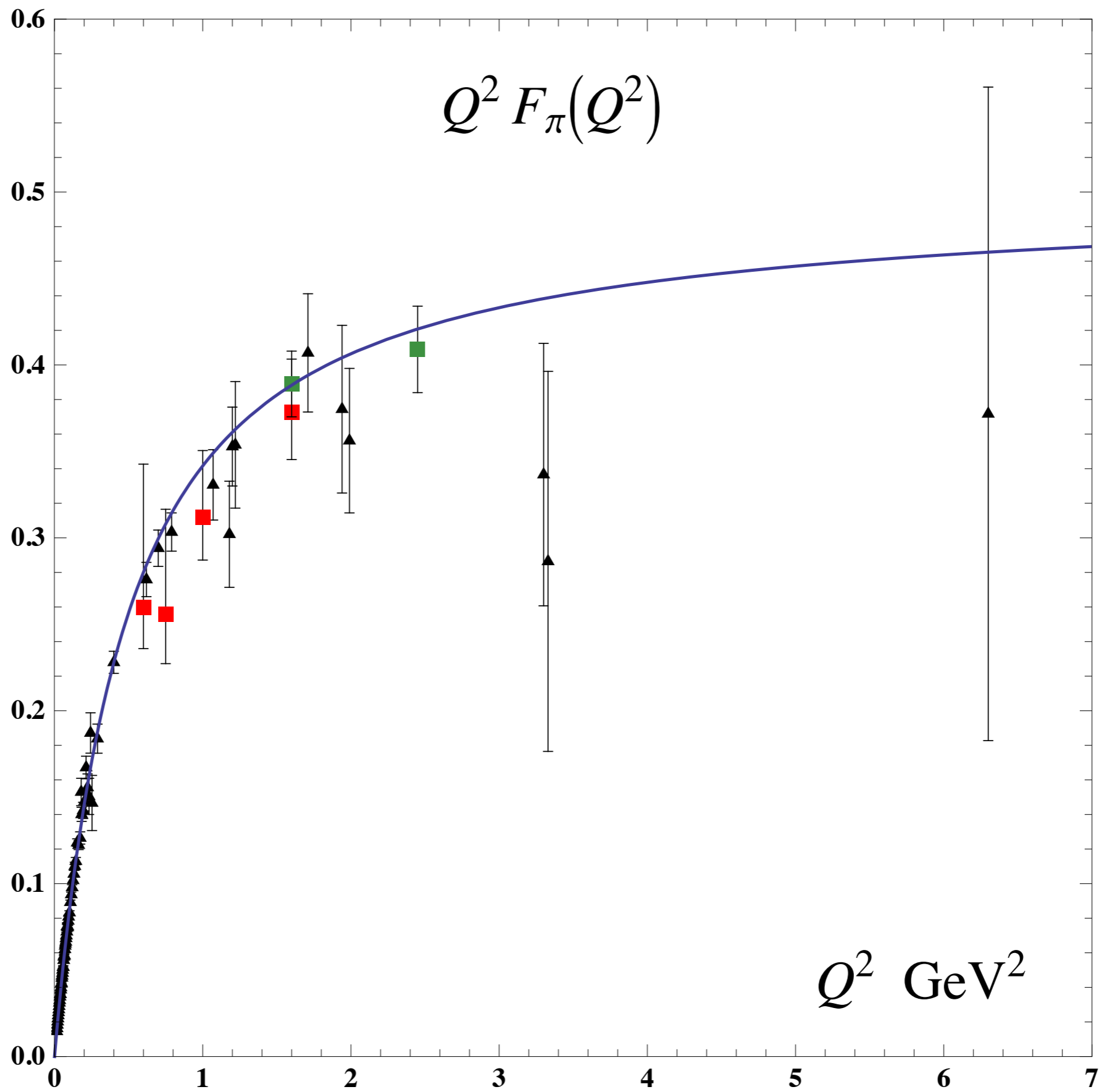
$\zeta = \sqrt{x(1-x)b_\perp^2} \longleftrightarrow z$

Fixed $\tau = t + z/c$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion



$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

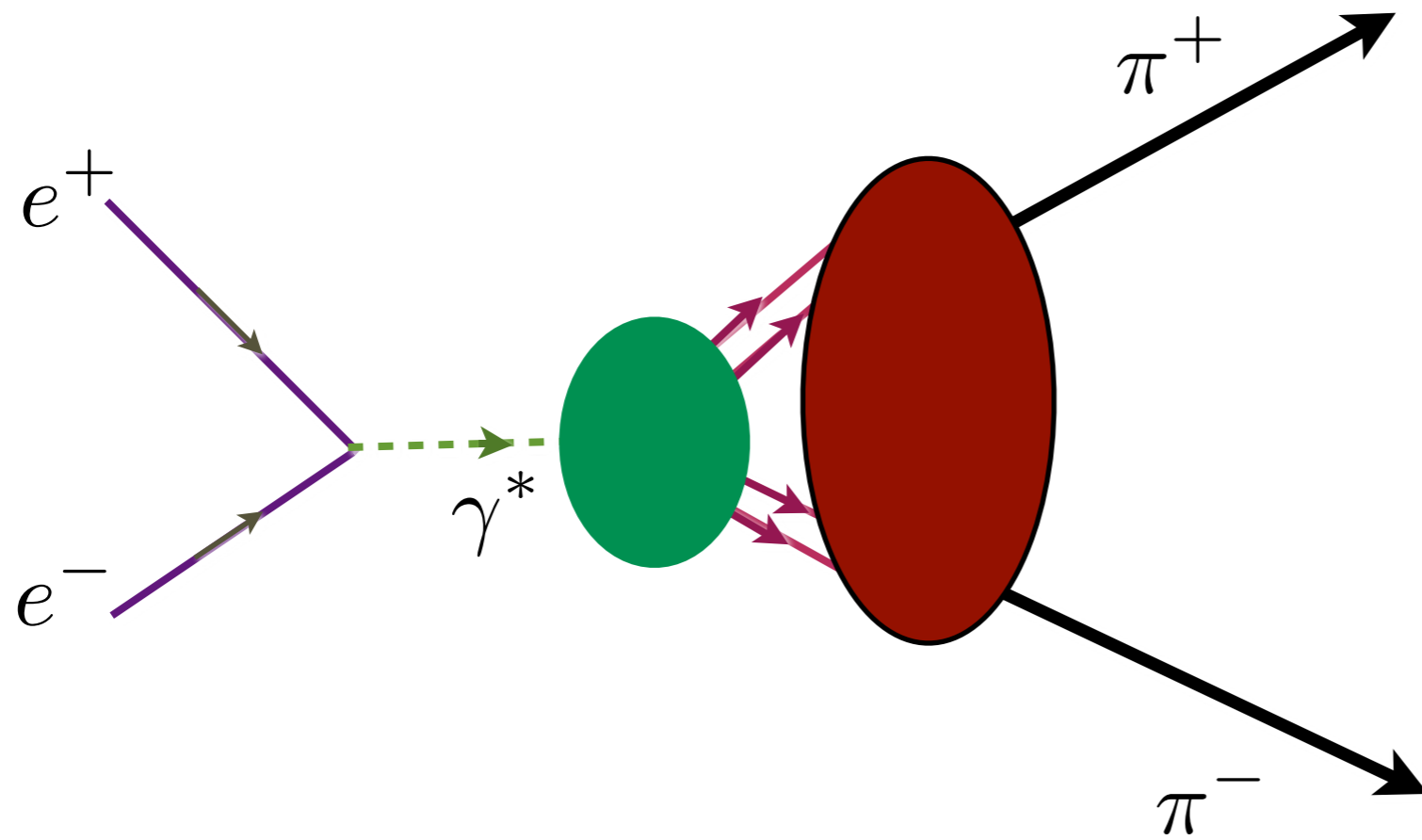
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

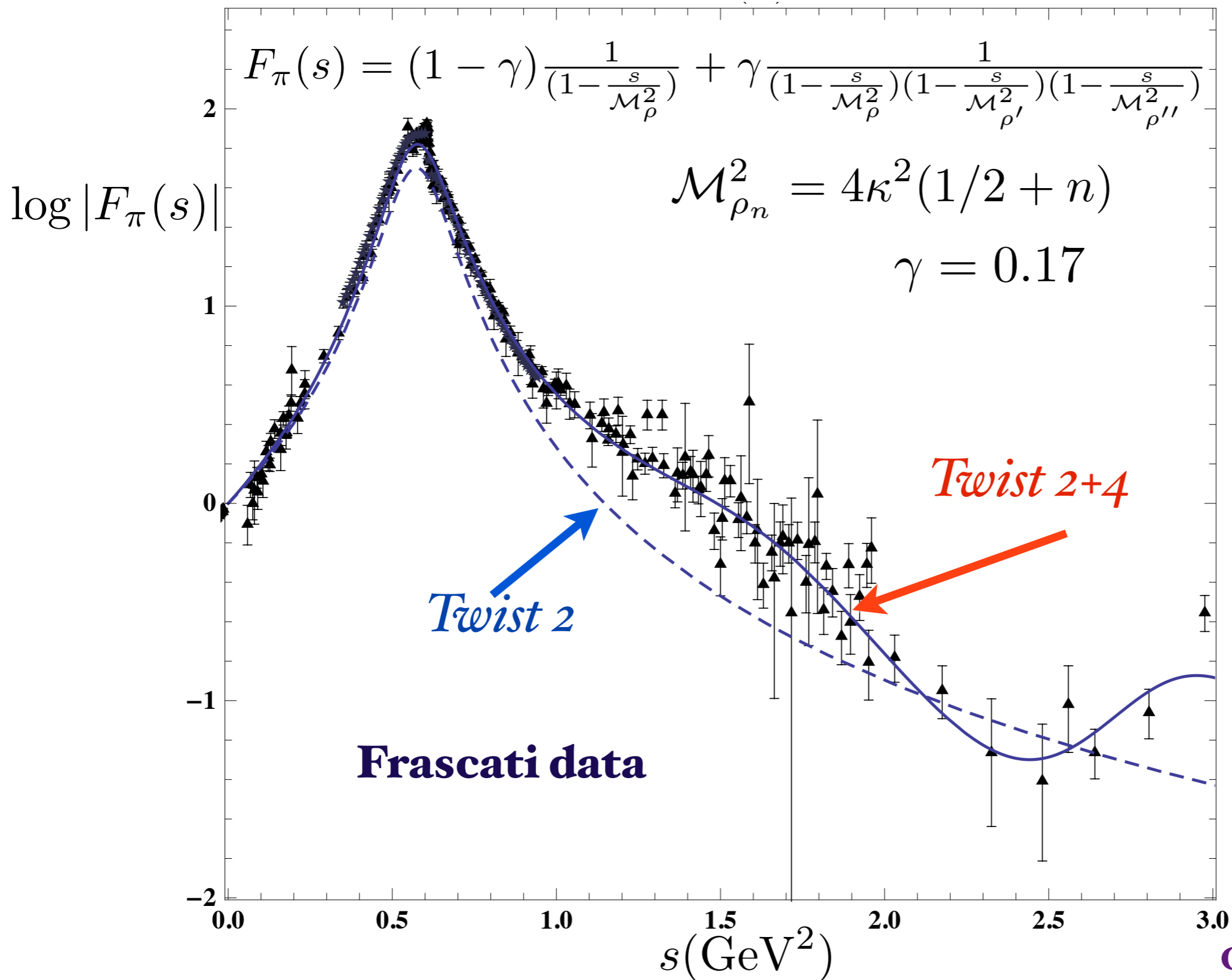
the external current decouples from the dilaton field.

*Dressed
Current
in Soft-Wall
Model*

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

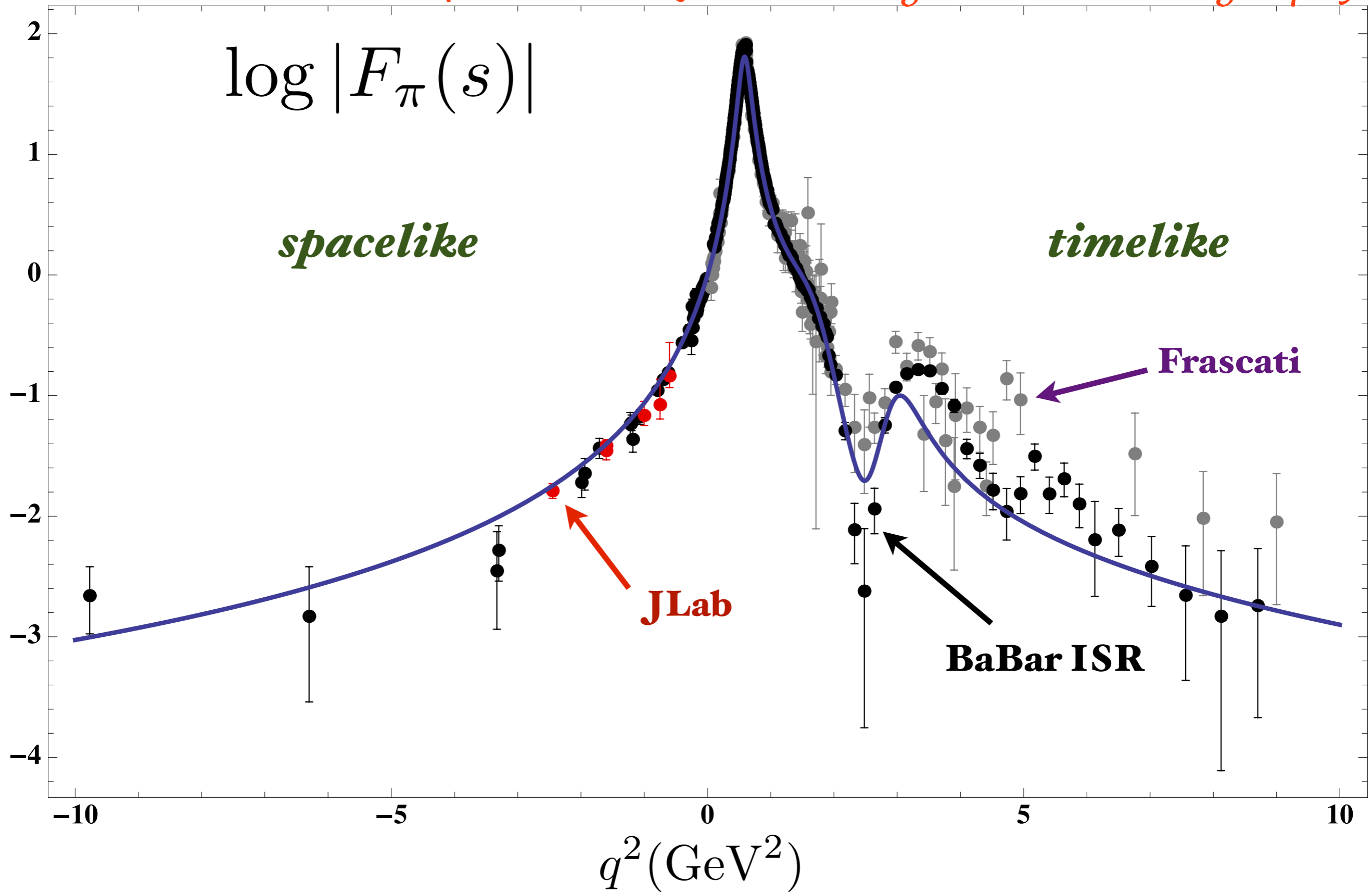


Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

Pion Form Factor from AdS/QCD and Light-Front Holography



Remarkable Features of Light-Front Schrödinger Equation

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$



Slovenia
July 2015

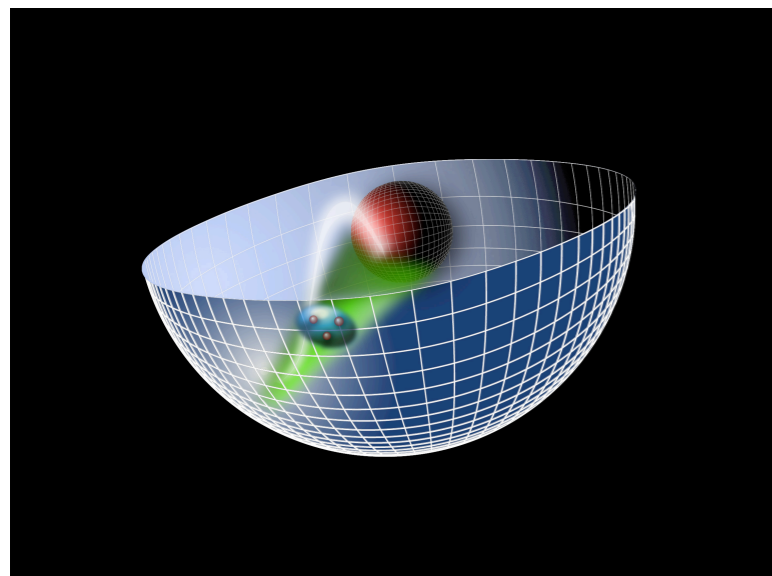
*New Perspectives for Hadron Physics
and the Cosmological Constant*

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY

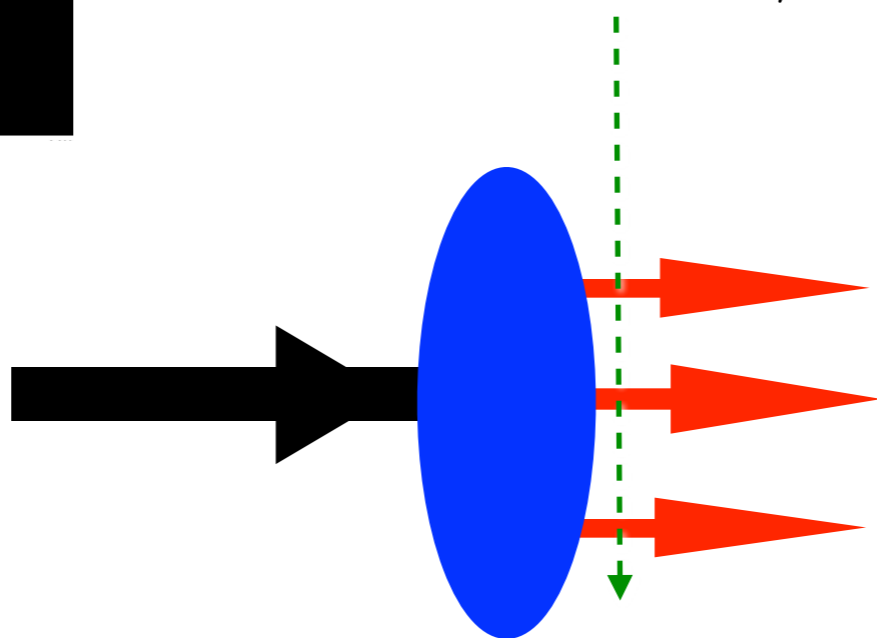
$$\phi(z)$$

AdS₅: Conformal Template for QCD

- *Light-Front Holography*

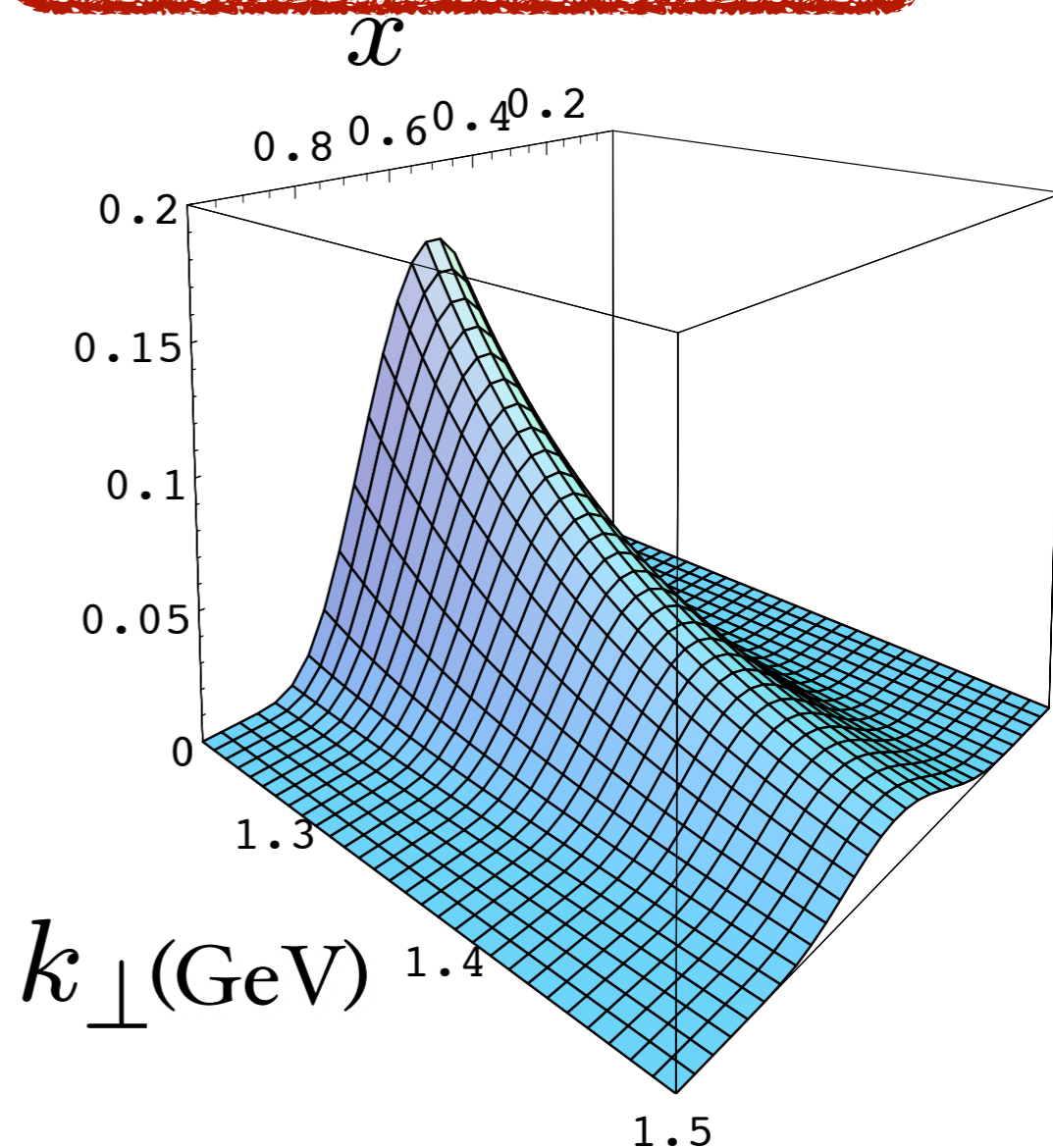


Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Duality of AdS₅ with LF Hamiltonian Theory



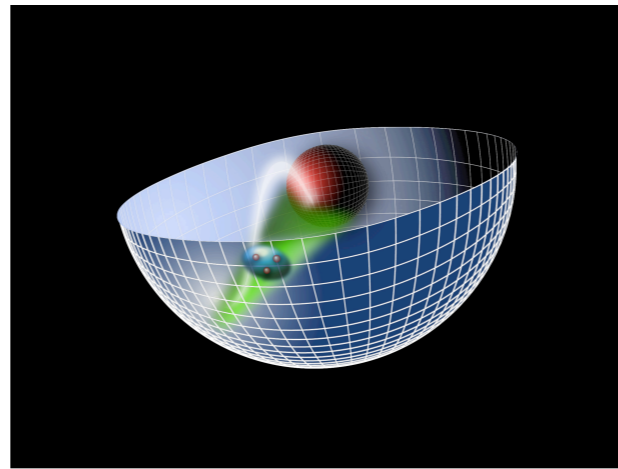
- *Light Front Wavefunctions:*

**Light-Front Schrödinger Equation
Spectroscopy and Dynamics**

AdS/QCD
Soft-Wall Model
 $e^{\varphi(z)} = e^{+\kappa^2 z^2}$

Single scheme-
independent fundamental
mass scale

κ



$$\zeta^2 = x(1-x)b_{\perp}^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

$$1/\kappa \simeq 1/3 \text{ fm}$$

Unique
Confinement Potential!
Conformal Symmetry
of the action

Confinement scale:
($\mathbf{m}_q=0$)

● de Alfaro, Fubini, Furlan:

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

Unique confinement potential!

● **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)\rangle = i\frac{\partial}{\partial\tau}|\psi(\tau)\rangle$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

● **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

What determines the QCD mass scale Λ_{QCD} ?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK \quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

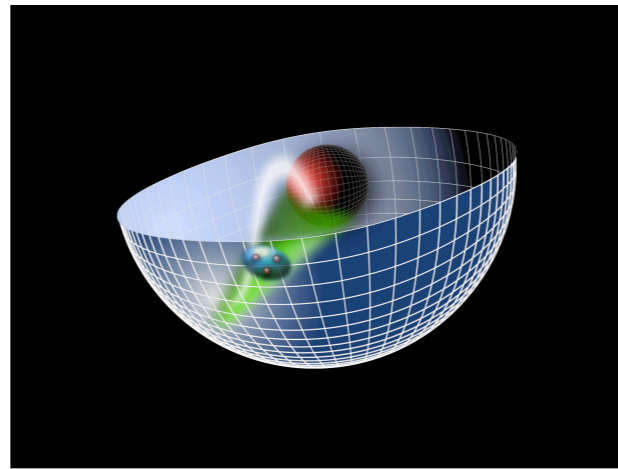


dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left(\frac{2tw + v}{\sqrt{4uw - v^2}} \right),$$

- **Identify with difference of LF time $\Delta x^+ / P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**





*AdS/QCD
Soft-Wall Model*

Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

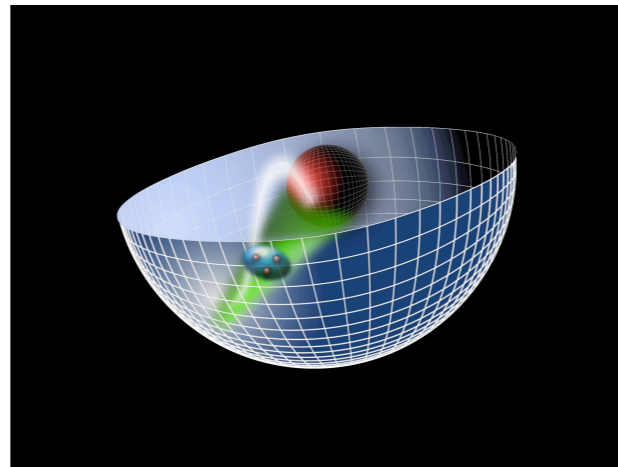
$$1/\kappa \simeq 1/3 \text{ fm}$$

Confinement scale:

***Unique
Confinement Potential!
Conformal Symmetry
of the action***

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**



*AdS/QCD
Soft-Wall Model*

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent

Unique color-confining potential

Zero mass pion for massless quarks

Regge trajectories with equal slopes in n and L

Light-Front Wavefunctions

Light-Front Schrödinger Equation

*Conformal Symmetry
of the action*

Interpretation of Mass Scale \mathcal{K}

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of \mathcal{K}
- Value of \mathcal{K} itself not determined -- place holder
- Need external constraint such as f_π
- “Zero-Parameter” Model

Baryon Spectrum in Soft-Wall Model

- Upon substitution $z \rightarrow \zeta$ and

$$\Psi_J(x, z) = e^{-iP \cdot x} z^2 \psi^J(z) u(P),$$

find LFWE for $d = 4$

$$\begin{aligned} \frac{d}{d\zeta} \psi_+^J + \frac{\nu + \frac{1}{2}}{\zeta} \psi_+^J + U(\zeta) \psi_+^J &= \mathcal{M} \psi_-^J, \\ -\frac{d}{d\zeta} \psi_-^J + \frac{\nu + \frac{1}{2}}{\zeta} \psi_-^J + U(\zeta) \psi_-^J &= \mathcal{M} \psi_+^J, \end{aligned}$$

$$U = \kappa^2 \zeta$$

- Eigenfunctions

$$\psi_+^J(\zeta) \sim \zeta^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \quad \psi_-^J(\zeta) \sim \zeta^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1), \quad \nu = L + 1 \quad (\tau = 3)$$

*Independent
of J*

- Full $J - L$ degeneracy (different J for same L) for baryons along given trajectory !



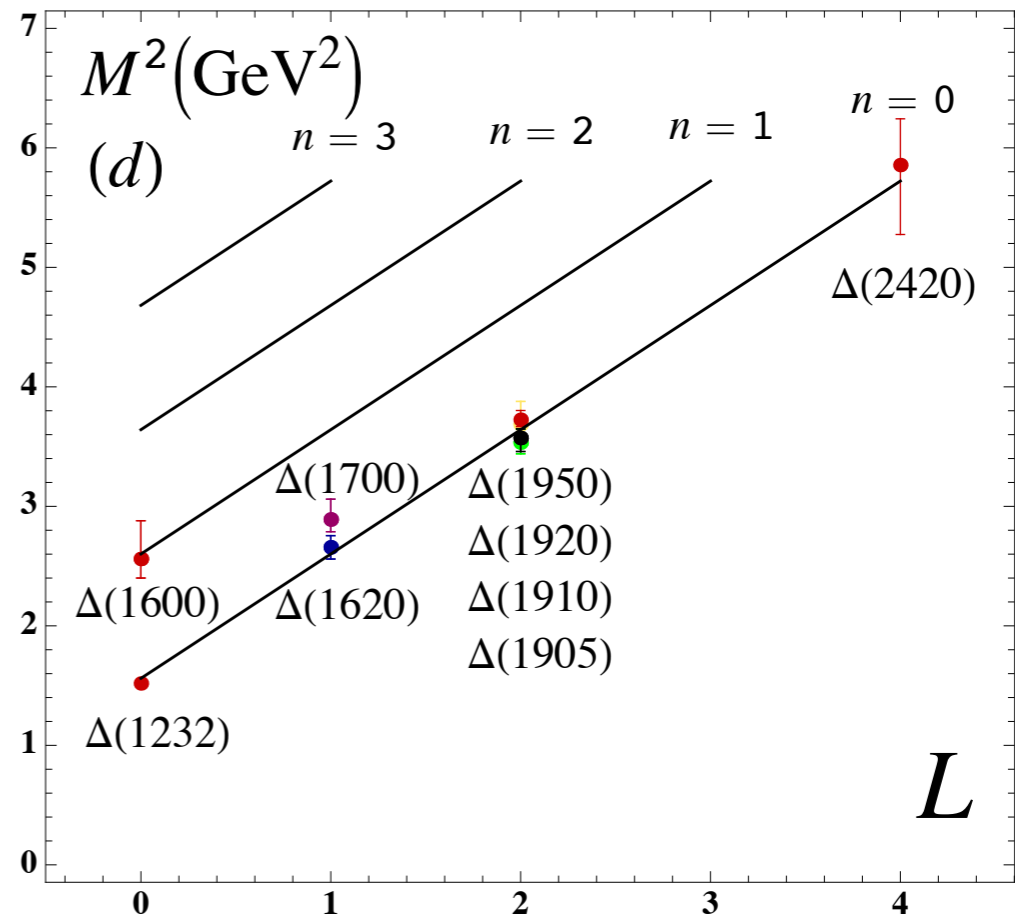
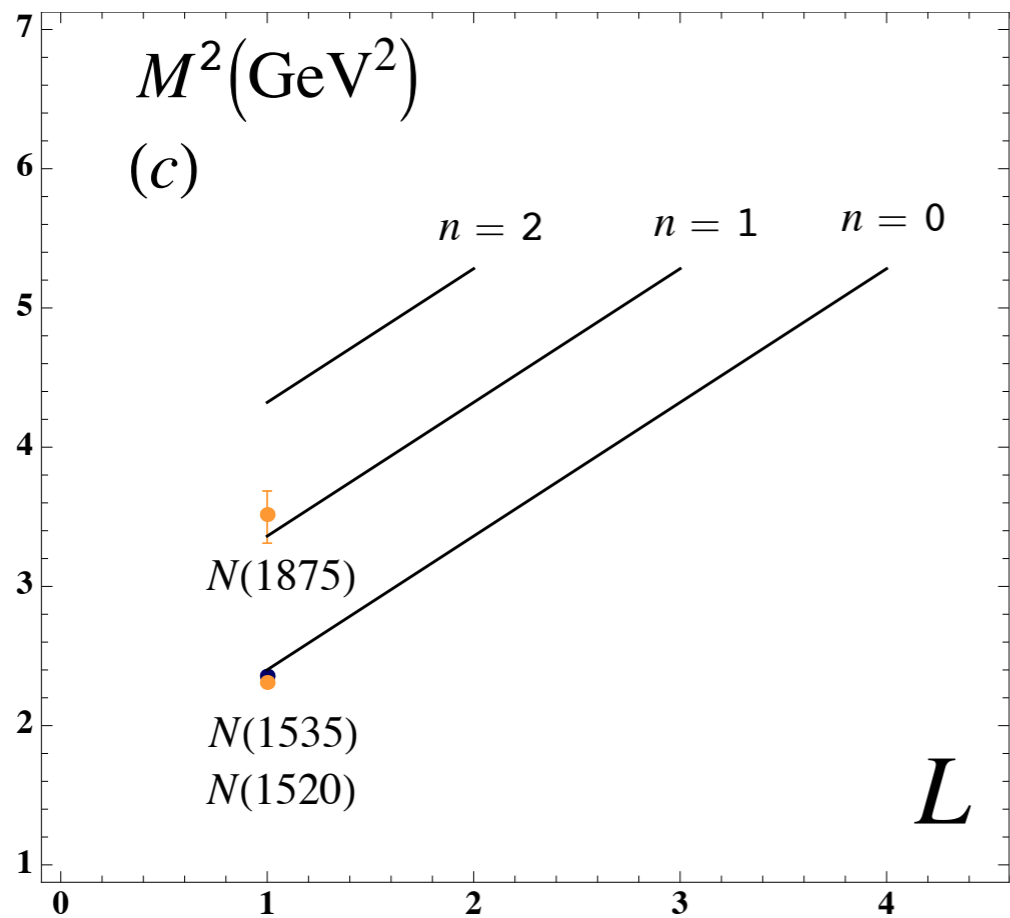
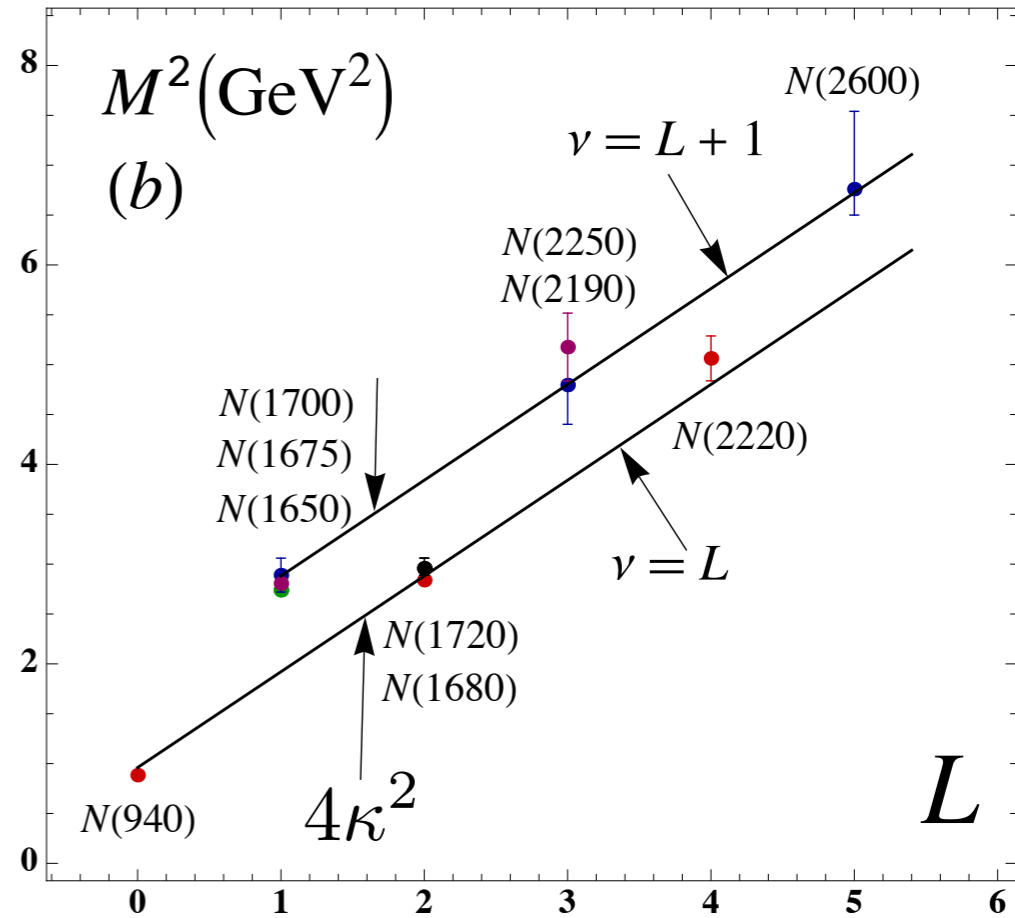
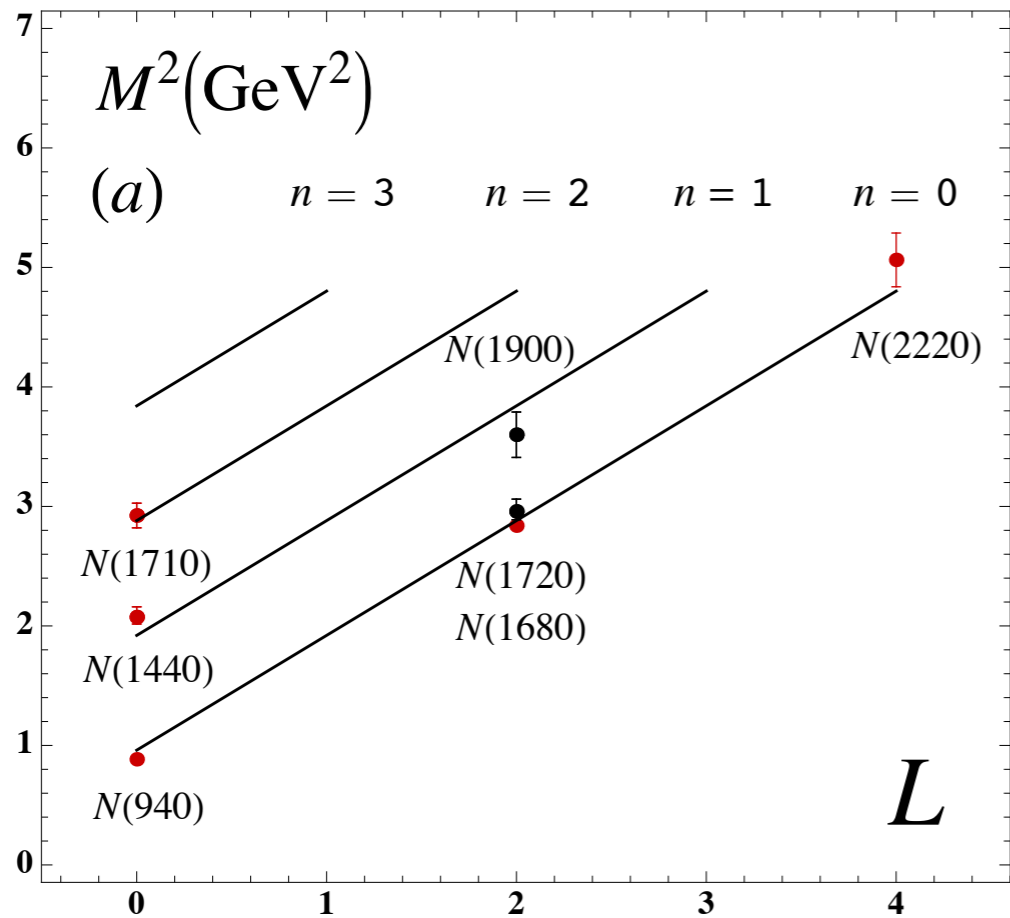


Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S , L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta_{\frac{5}{2}}^{-}(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

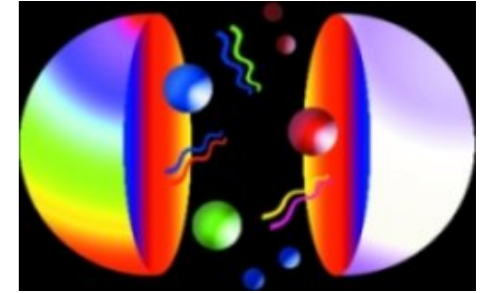
$SU(6)$	S	L	n	Baryon State				
56	$\frac{1}{2}$	0	0	$N_{\frac{1}{2}}^{1+}(940)$				
	$\frac{1}{2}$	0	1	$N_{\frac{1}{2}}^{1+}(1440)$				
	$\frac{1}{2}$	0	2	$N_{\frac{1}{2}}^{1+}(1710)$				
	$\frac{3}{2}$	0	0	$\Delta_{\frac{3}{2}}^{3+}(1232)$				
	$\frac{3}{2}$	0	1	$\Delta_{\frac{3}{2}}^{3+}(1600)$				
70	$\frac{1}{2}$	1	0	$N_{\frac{1}{2}}^{1-}(1535) \quad N_{\frac{3}{2}}^{3-}(1520)$				
	$\frac{3}{2}$	1	0	$N_{\frac{1}{2}}^{1-}(1650)$	$N_{\frac{3}{2}}^{3-}(1700)$	$N_{\frac{5}{2}}^{5-}(1675)$		
	$\frac{3}{2}$	1	1	$N_{\frac{1}{2}}^{1-}$	$N_{\frac{3}{2}}^{3-}(1875)$	$N_{\frac{5}{2}}^{5-}$		
	$\frac{1}{2}$	1	0	$\Delta_{\frac{1}{2}}^{1-}(1620) \quad \Delta_{\frac{3}{2}}^{3-}(1700)$				
56	$\frac{1}{2}$	2	0	$N_{\frac{3}{2}}^{3+}(1720) \quad N_{\frac{5}{2}}^{5+}(1680)$				
	$\frac{1}{2}$	2	1	$N_{\frac{3}{2}}^{3+}(1900) \quad N_{\frac{5}{2}}^{5+}$				
	$\frac{3}{2}$	2	0	$\Delta_{\frac{1}{2}}^{1+}(1910)$	$\Delta_{\frac{3}{2}}^{3+}(1920)$	$\Delta_{\frac{5}{2}}^{5+}(1905)$	$\Delta_{\frac{7}{2}}^{7+}(1950)$	
70	$\frac{1}{2}$	3	0	$N_{\frac{5}{2}}^{5-} \quad N_{\frac{7}{2}}^{7-}$				
	$\frac{3}{2}$	3	0	$N_{\frac{3}{2}}^{3-}$	$N_{\frac{5}{2}}^{5-}$	$N_{\frac{7}{2}}^{7-}(2190)$	$N_{\frac{9}{2}}^{9-}(2250)$	
	$\frac{1}{2}$	3	0	$\Delta_{\frac{5}{2}}^{5-} \quad \Delta_{\frac{7}{2}}^{7-}$				
56	$\frac{1}{2}$	4	0	$N_{\frac{7}{2}}^{7+} \quad N_{\frac{9}{2}}^{9+}(2220)$				
	$\frac{3}{2}$	4	0	$\Delta_{\frac{5}{2}}^{5+}$	$\Delta_{\frac{7}{2}}^{7+}$	$\Delta_{\frac{9}{2}}^{9+}$	$\Delta_{\frac{11}{2}}^{11+}(2420)$	
70	$\frac{1}{2}$	5	0	$N_{\frac{9}{2}}^{9-} \quad N_{\frac{11}{2}}^{11-}$				
	$\frac{3}{2}$	5	0	$N_{\frac{7}{2}}^{7-}$	$N_{\frac{9}{2}}^{9-}$	$N_{\frac{11}{2}}^{11-}(2600)$	$N_{\frac{13}{2}}^{13-}$	

PDG 2012

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

*Chiral Symmetry
of Eigenstate!*

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$
 $J^z = +1/2 : \langle L^z \rangle = 1/2, \langle S_q^z \rangle = 0$
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$.**

No mass-degenerate parity partners!

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}(\kappa^2 z^2) e^{-\kappa^2 z^2/2}$$

- Normalization ($F_1^p(0) = 1$, $V(Q=0, z) = 1$)

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

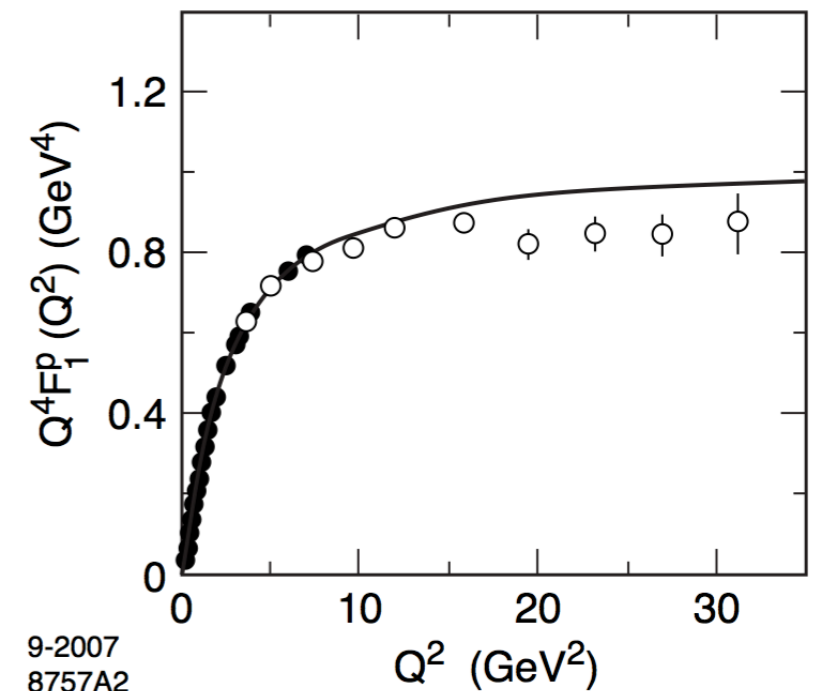
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$



1+1

$$\{\psi, \psi^+\} = 1$$

*two anti-commuting
fermionic operators*

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \quad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Superconformal Algebra

$$\{\psi, \psi^+\} = 1 \quad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \quad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates the conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

Superconformal Algebra

Baryon Equation

Consider $R_w = Q + wS$; w : dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \quad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B$, $w = \kappa^2$

Eigenvalue of G : $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

$$x \rightarrow \zeta$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+, \quad \text{G}_{22}$$
$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^-. \quad \text{G}_{11}$$

$$M_B^2(N, L_B) = 4\lambda_B(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

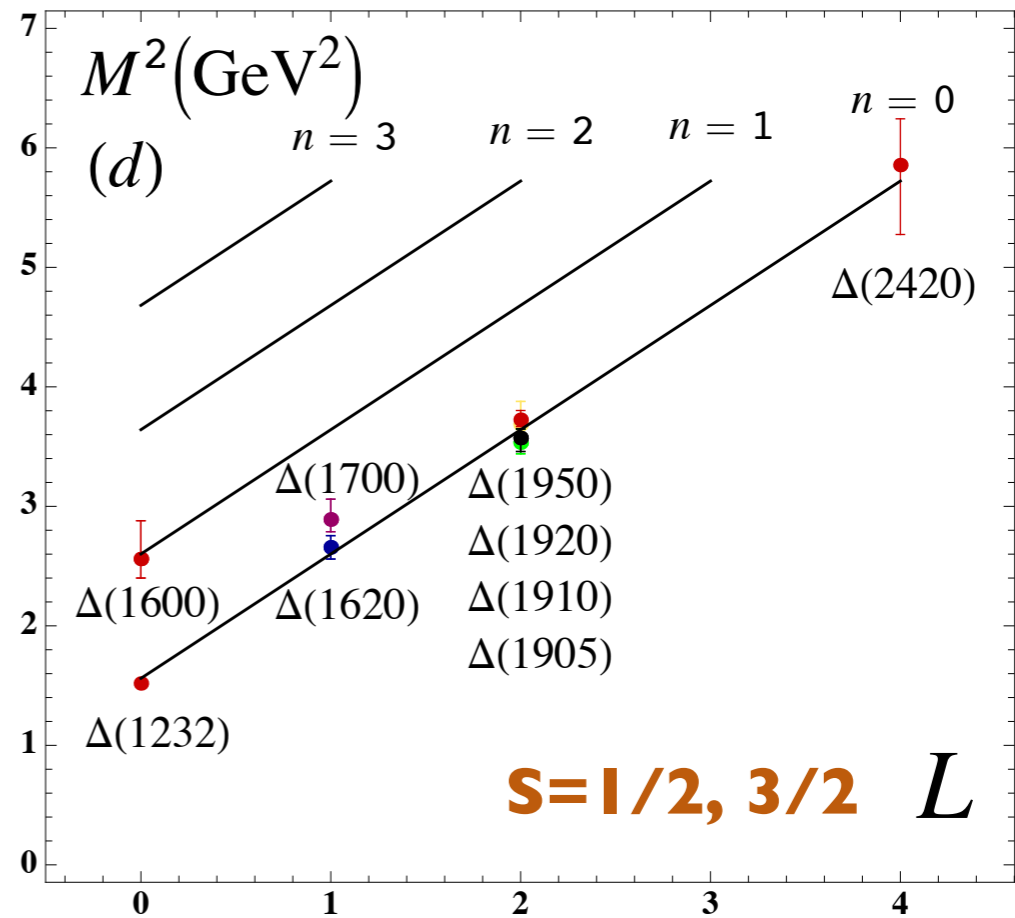
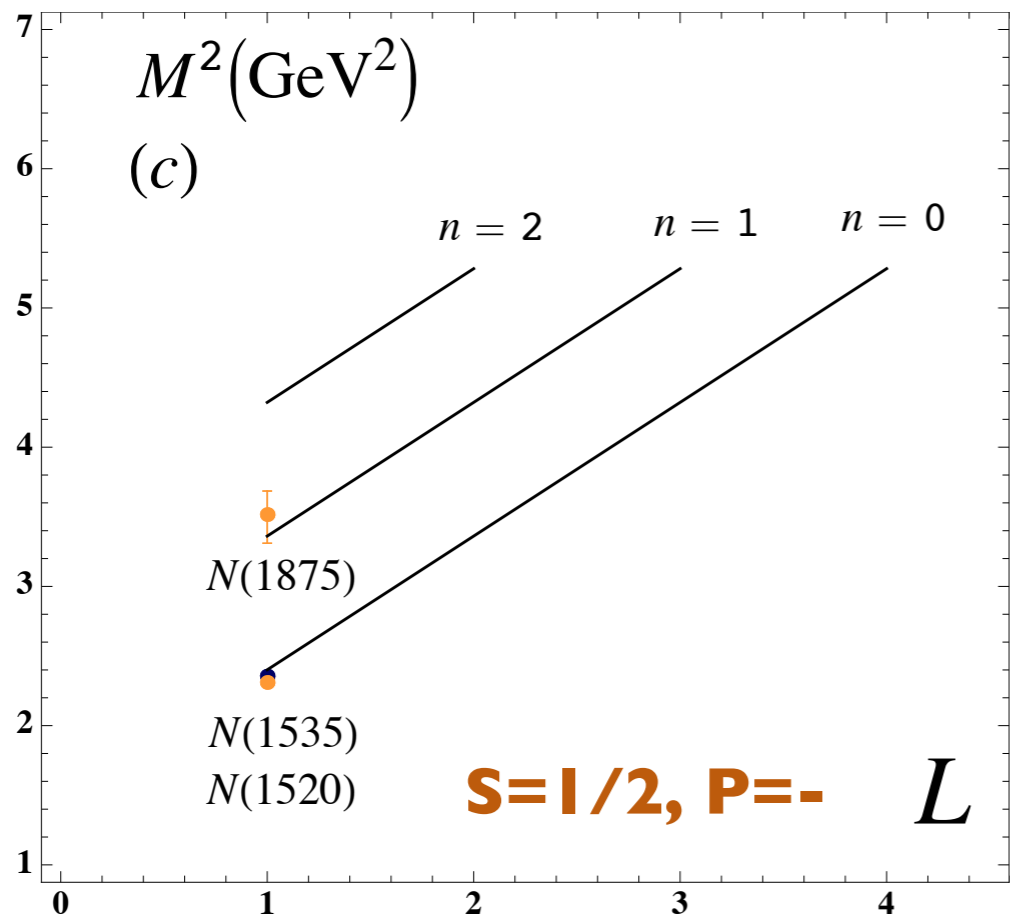
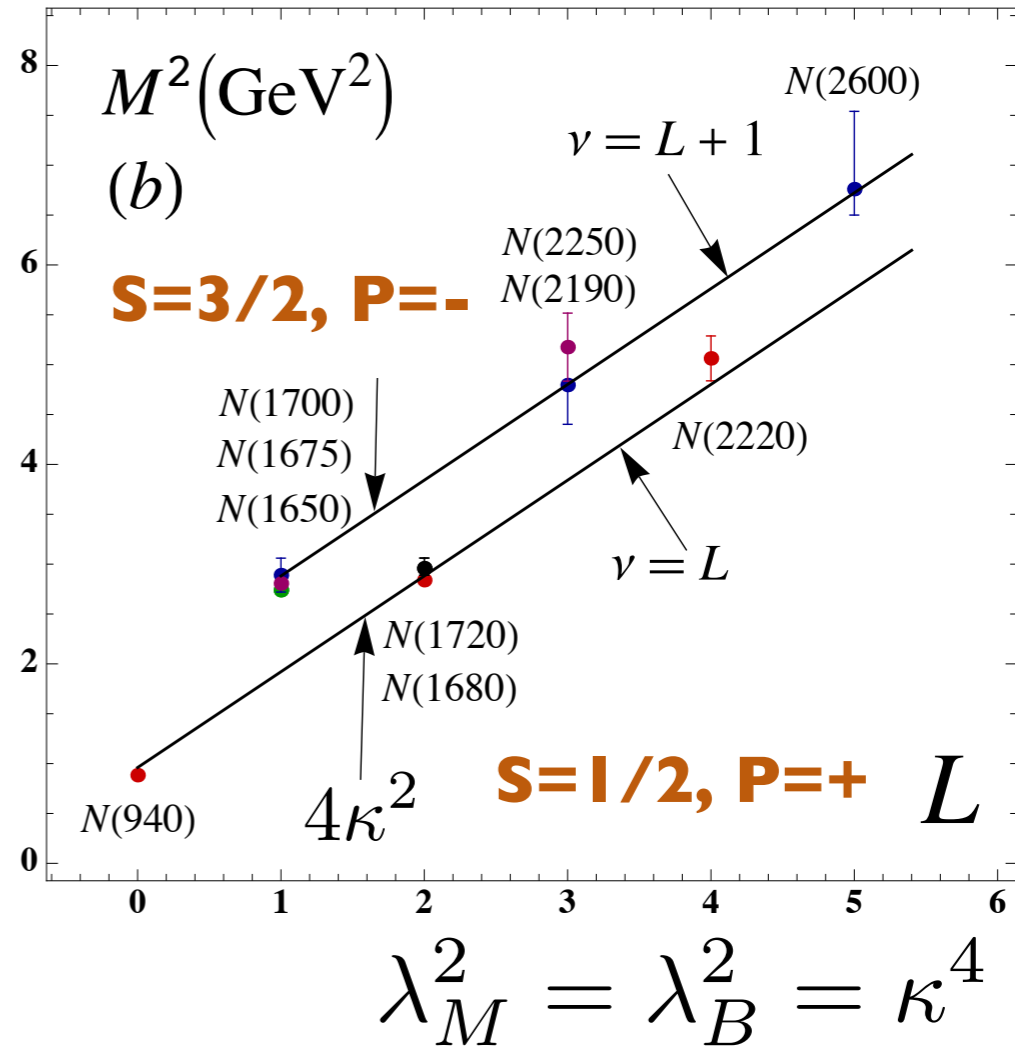
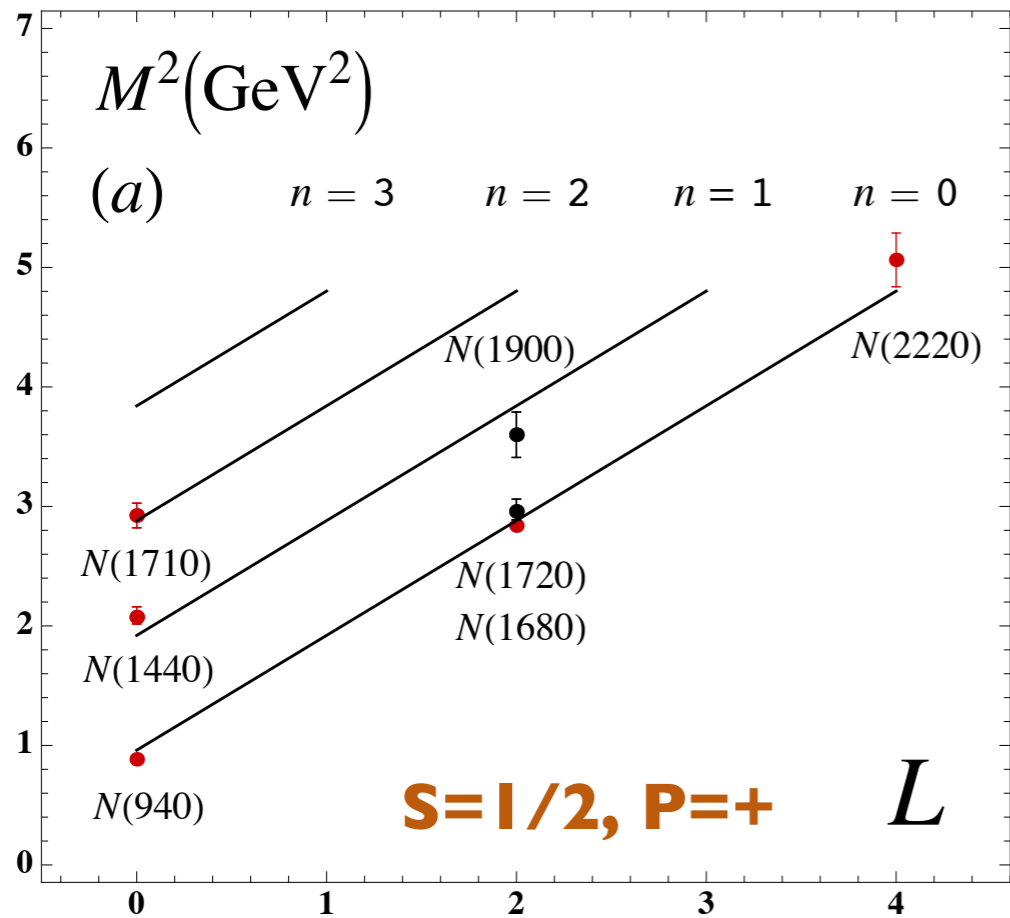
$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_j \quad \text{G}_{11}$$

$$M_M^2(N, L_M, S = 0) = 4\lambda_M(n + L_M)$$

S=0, I=I Meson is superpartner of S=1/2, I=I Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

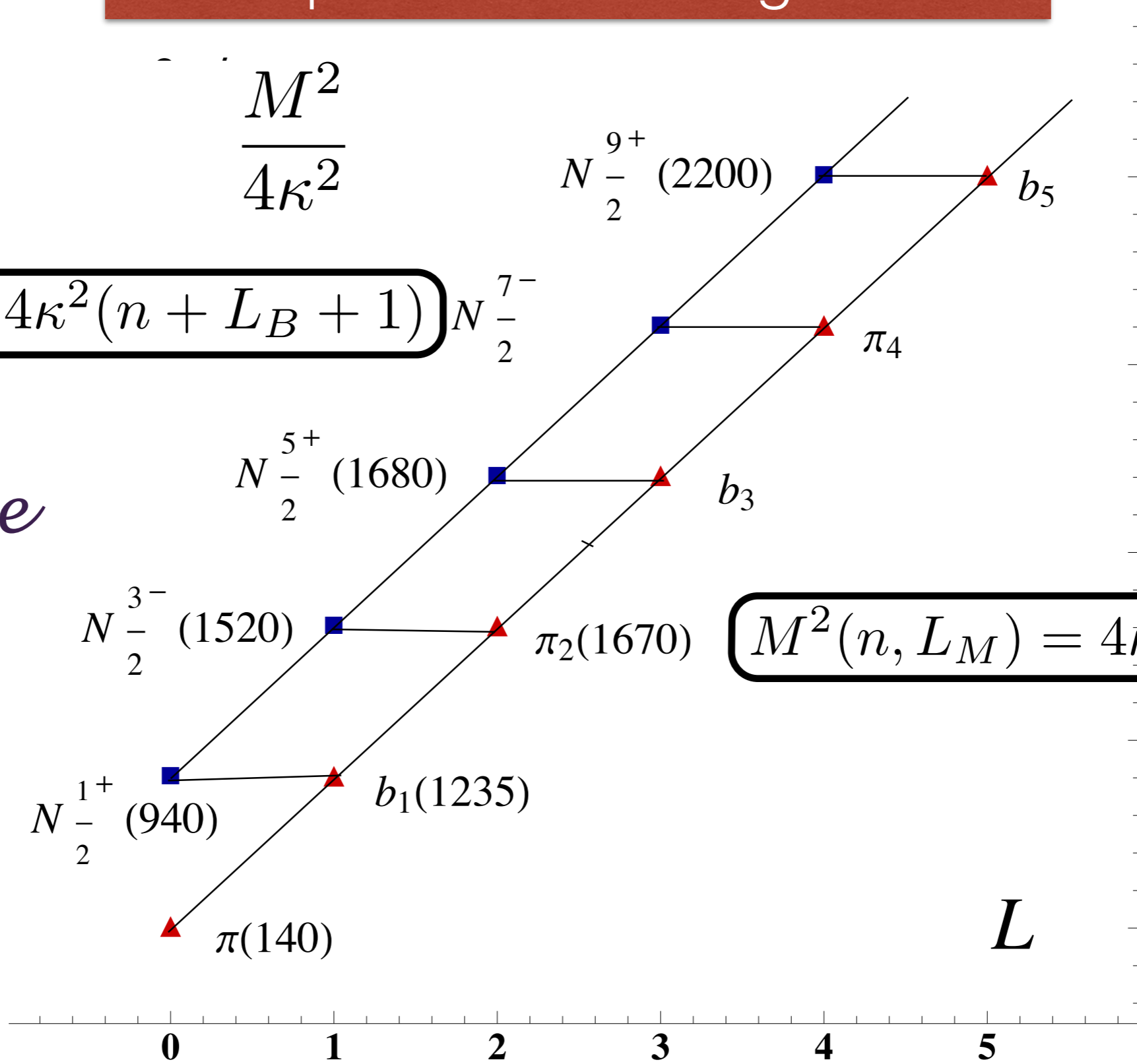
$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$



Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



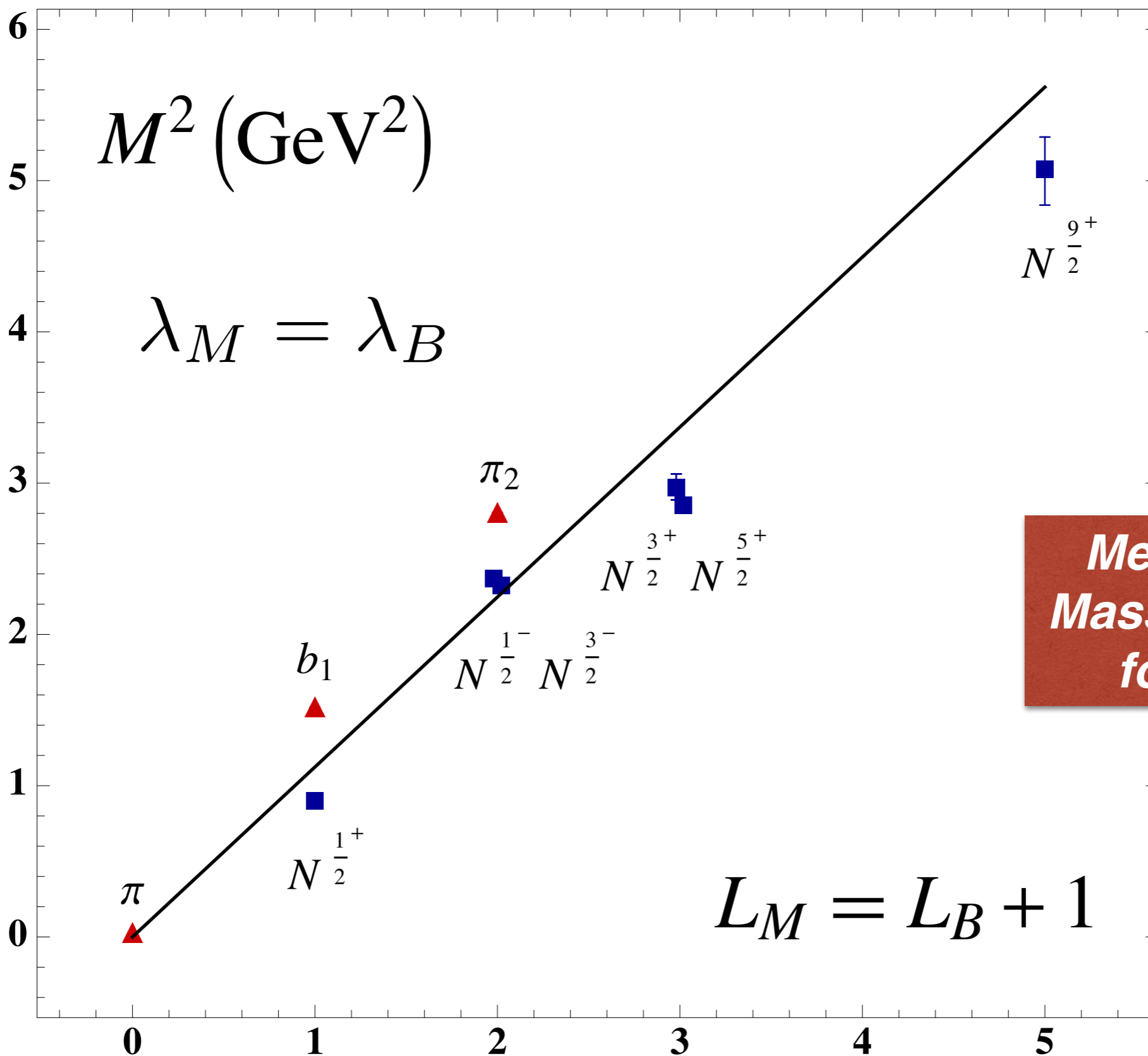
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

**Meson-Baryon
Mass Degeneracy
for $L_M=L_B+1$**

$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$

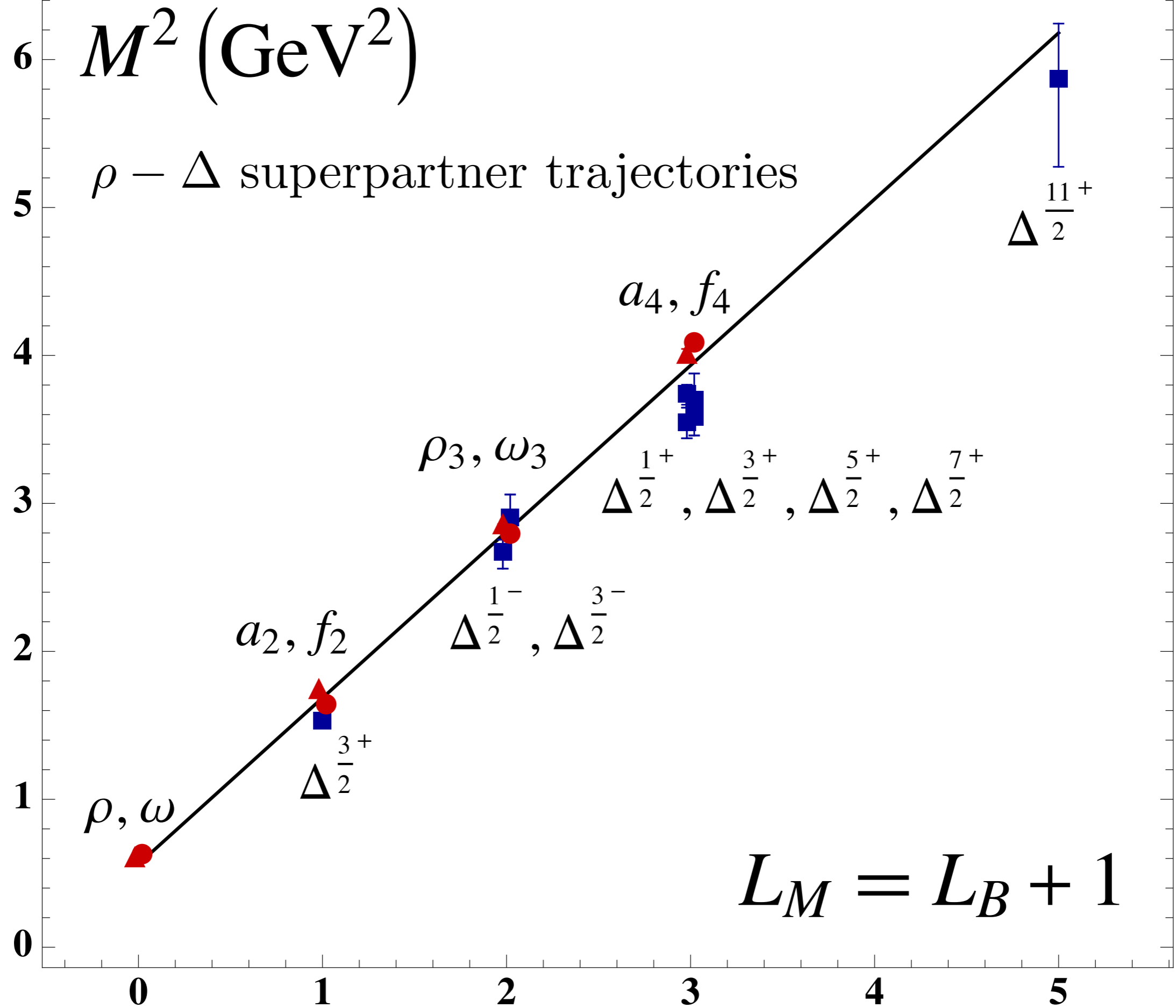
**Superconformal AdS Light-Front Holographic
QCD (LFHQCD):
Identical meson and baryon spectra!**

$$\lambda = \kappa^2$$



M^2 (GeV²)

$\rho - \Delta$ superpartner trajectories



Features of Supersymmetric Equations

- $J = L + S$ baryon simultaneously satisfies both equations of G with L , $L + 1$ for same mass eigenvalue
- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $S^z = \pm 1/2$
- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = L^z + 1/2$
- Mass-degenerate meson “superpartner” with $L_M = L_B + 1$. *“Shifted meson-baryon Duality”*

Meson and baryon have same κ !

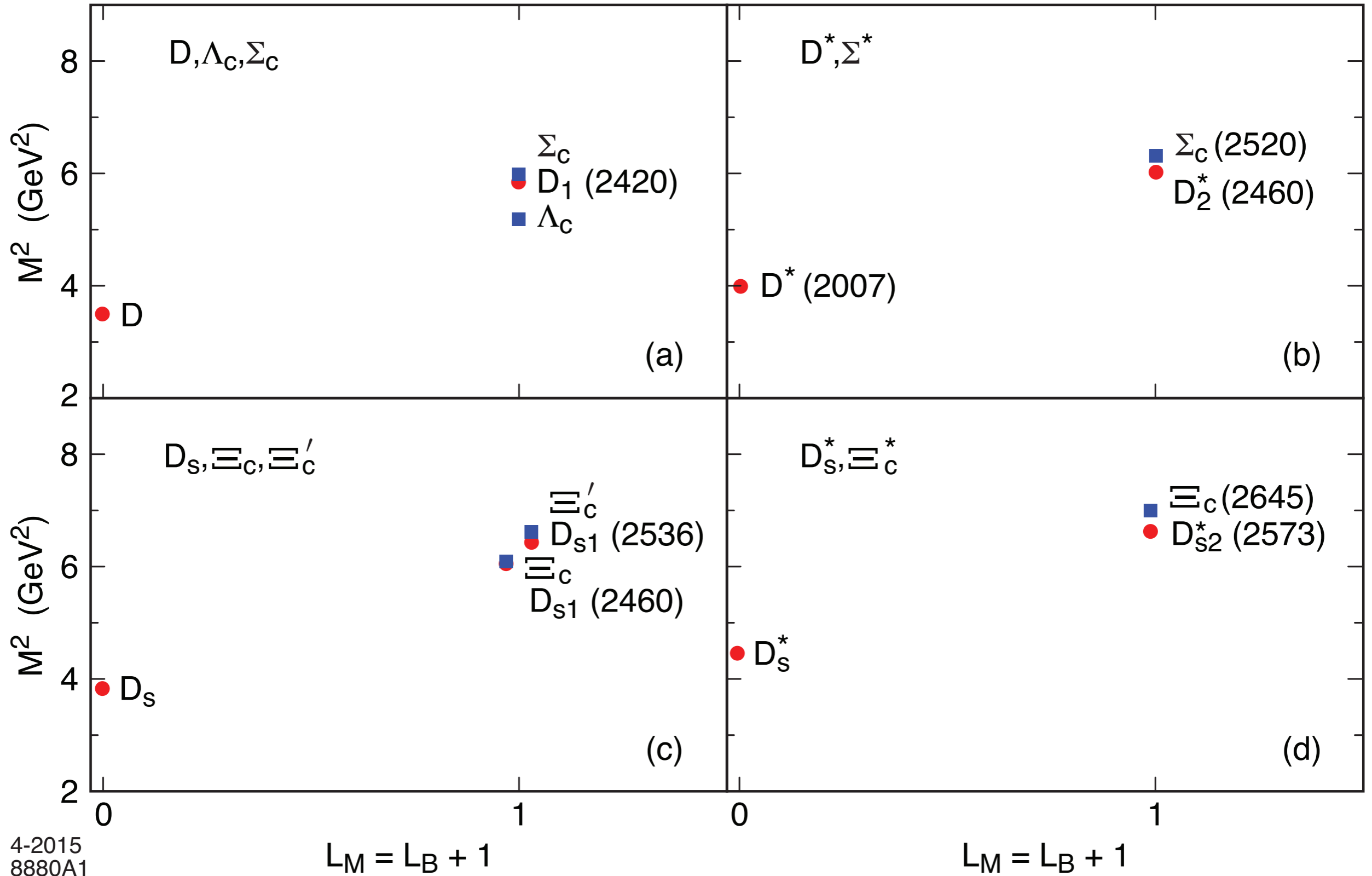


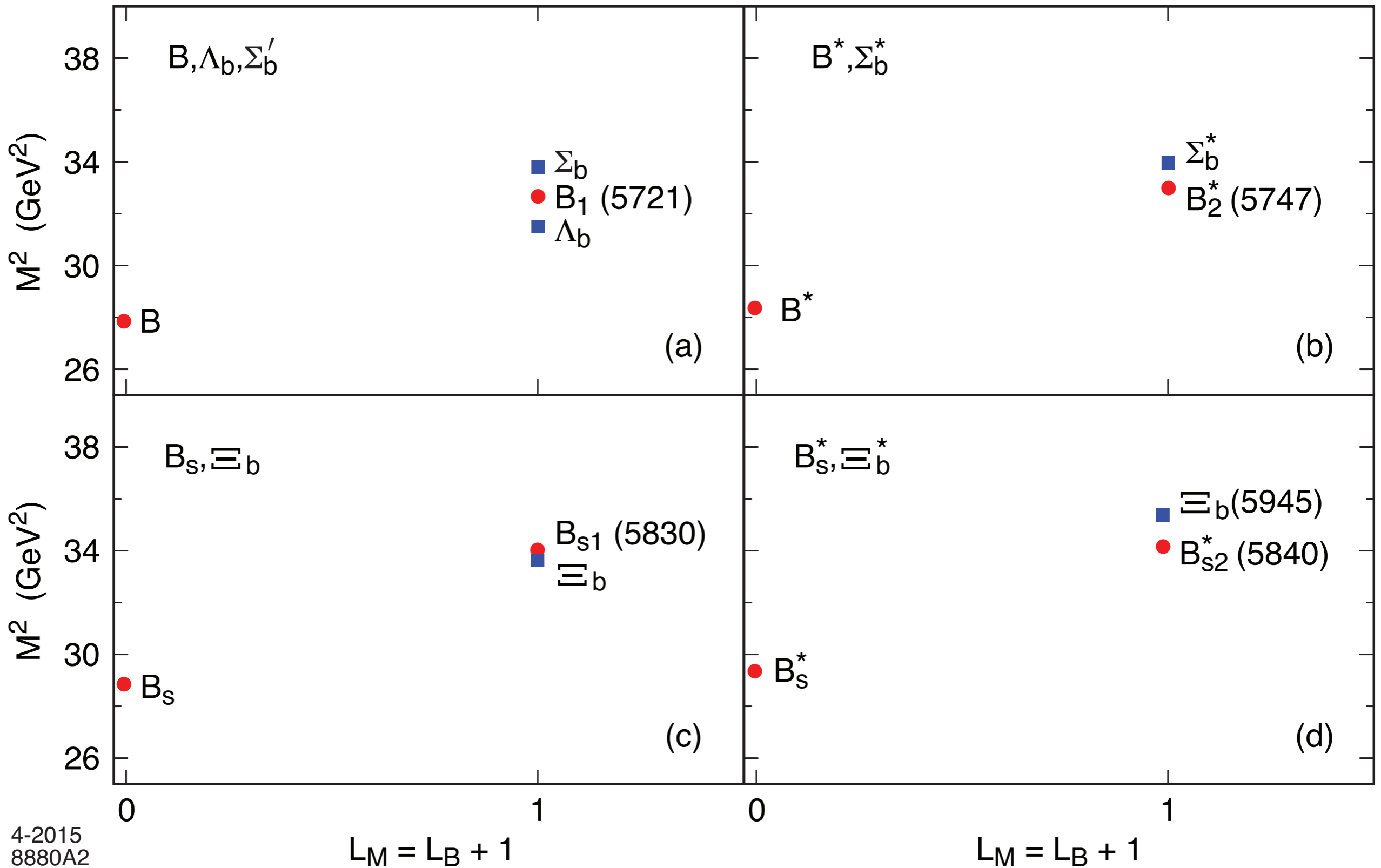
Supersymmetry Across the Light and Heavy-Light Hadronic Spectrum

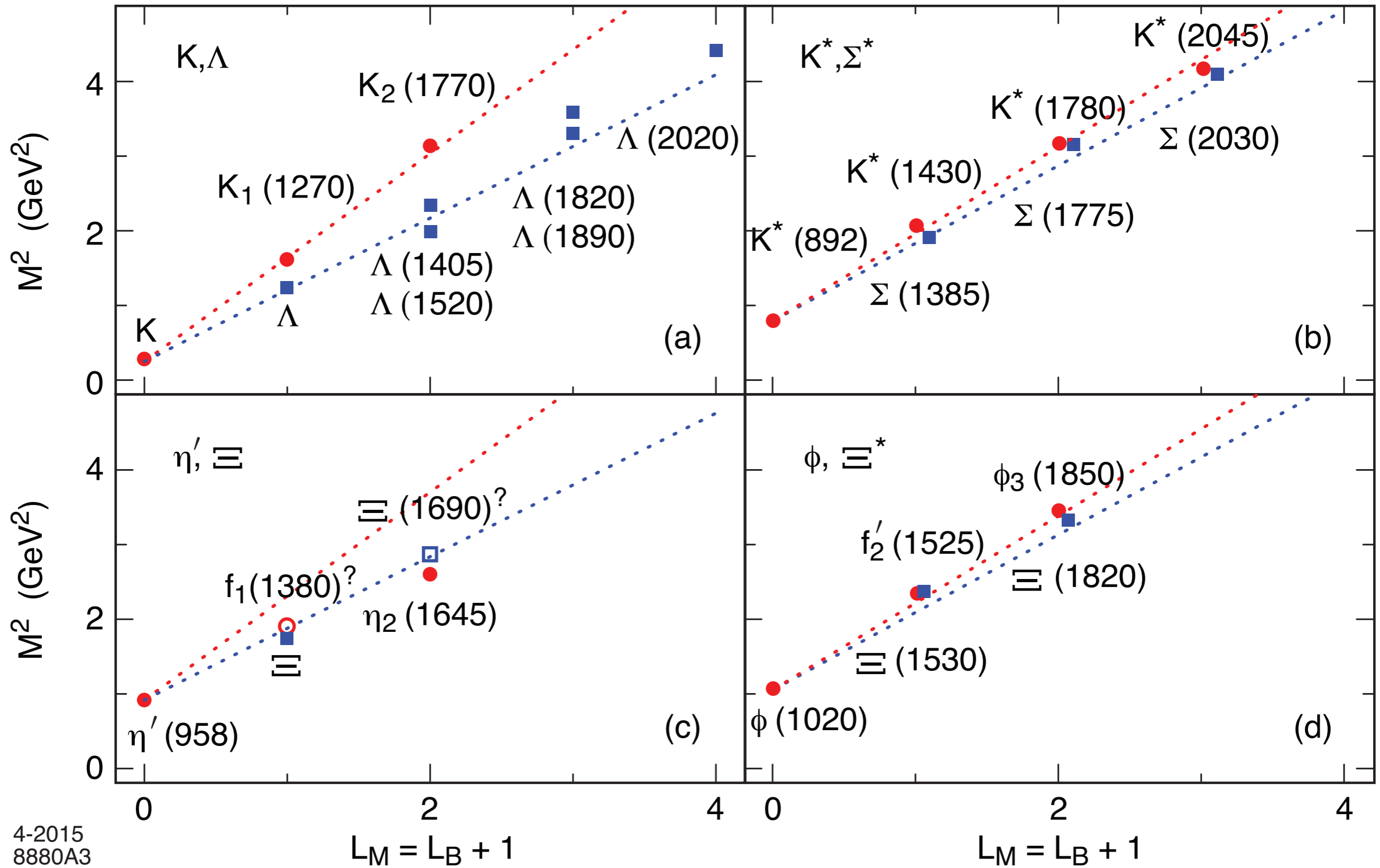
Dosch, de Teramond, sjb

Relativistic light-front bound-state equations for mesons and baryons can be constructed in the chiral limit from the supercharges of a superconformal algebra which connect baryon and meson spectra. Quark masses break the conformal invariance, but the basic underlying supersymmetric mechanism, which transforms meson and baryon wave functions into each other, still holds and gives remarkable connections across the entire spectrum of light and heavy-light hadrons. We also briefly examine the consequences of extending the supersymmetric relations to double-heavy mesons and baryons.

Dosch, de Teramond, sjb







Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

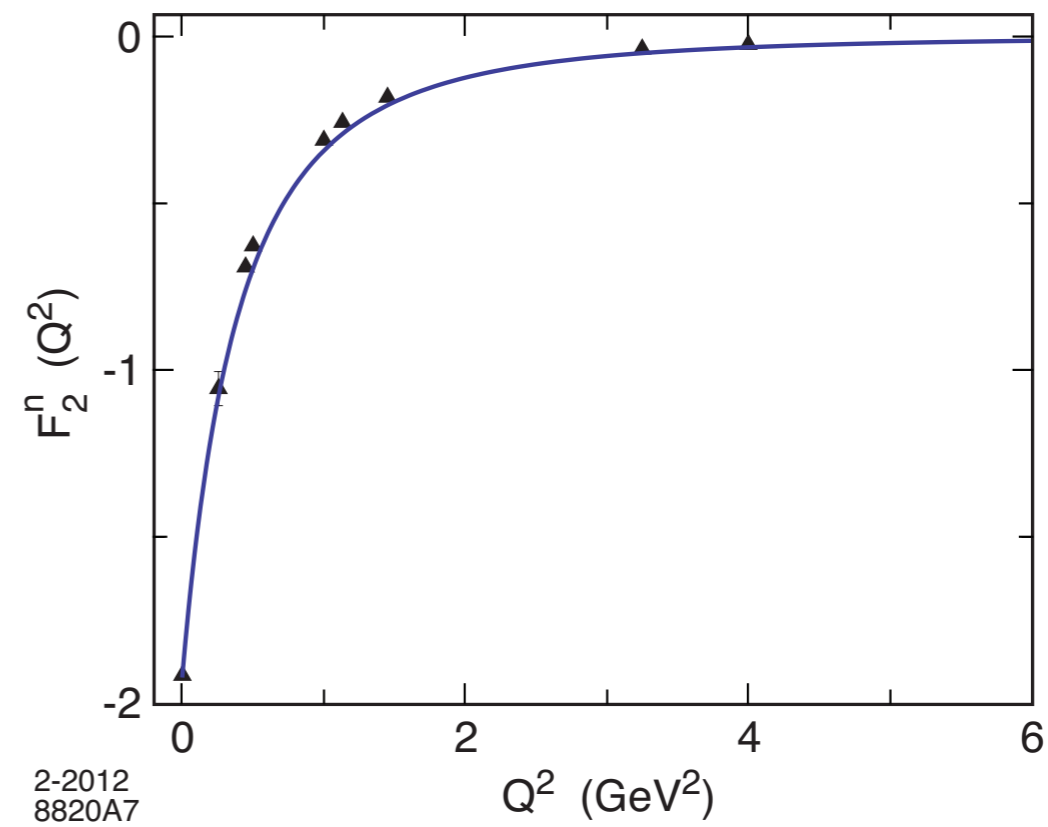
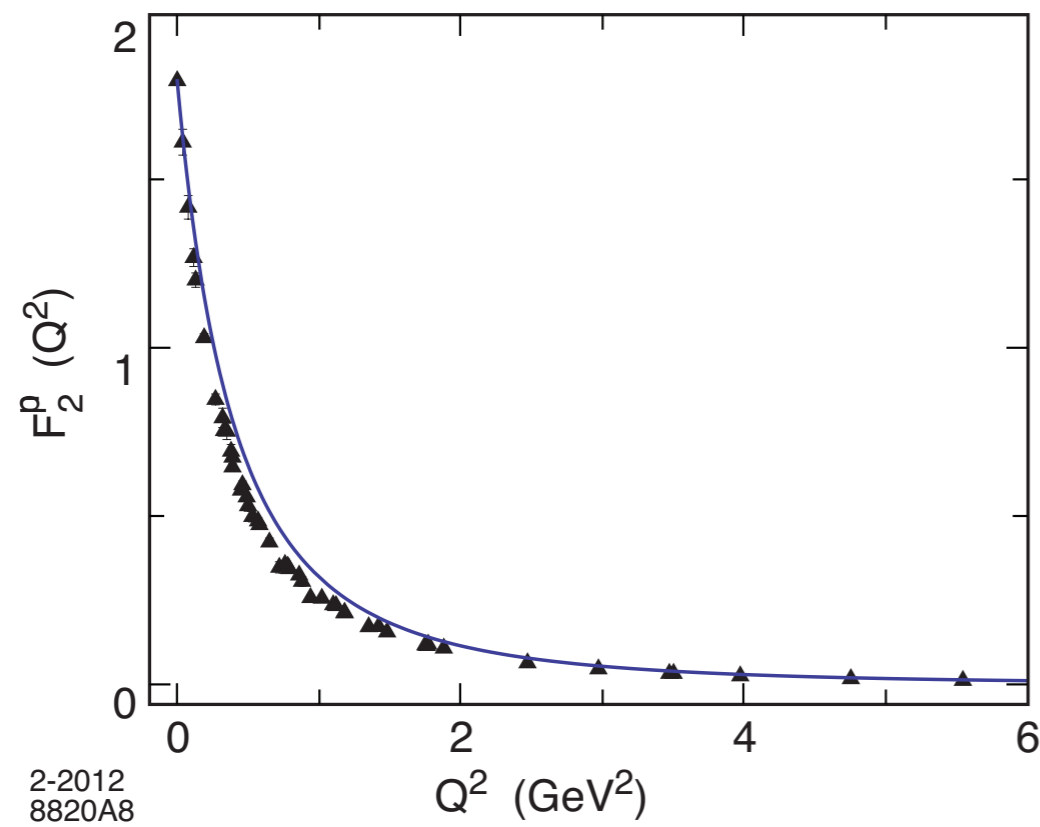
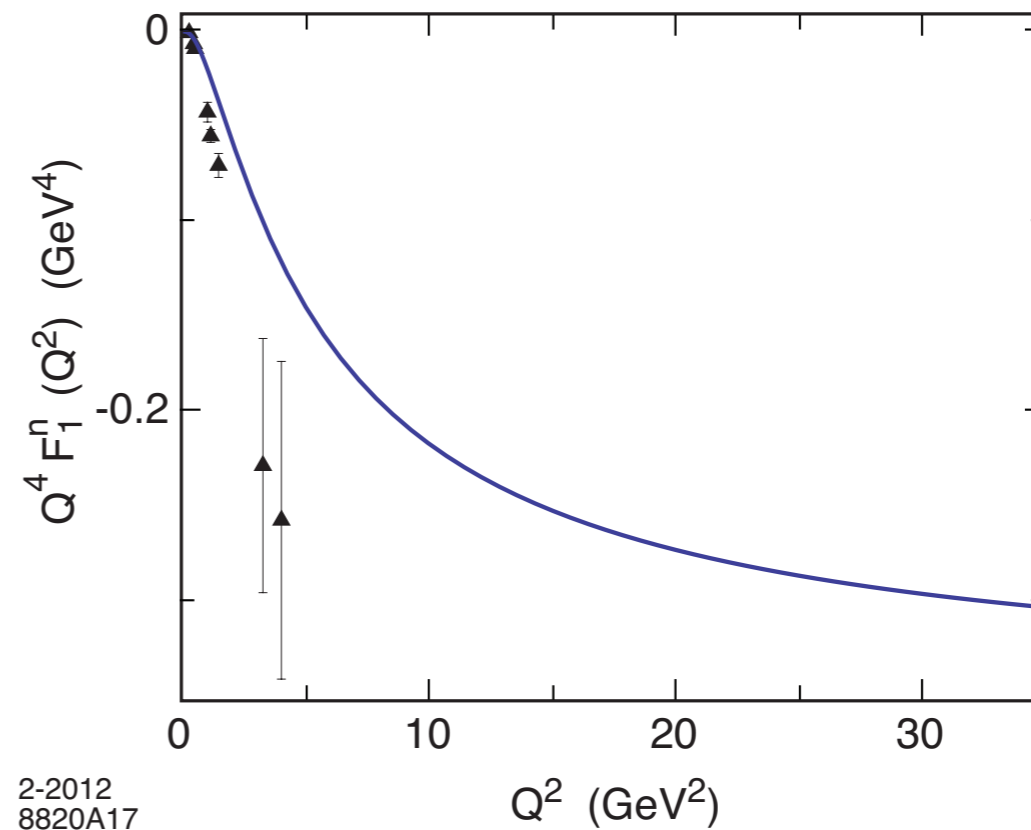
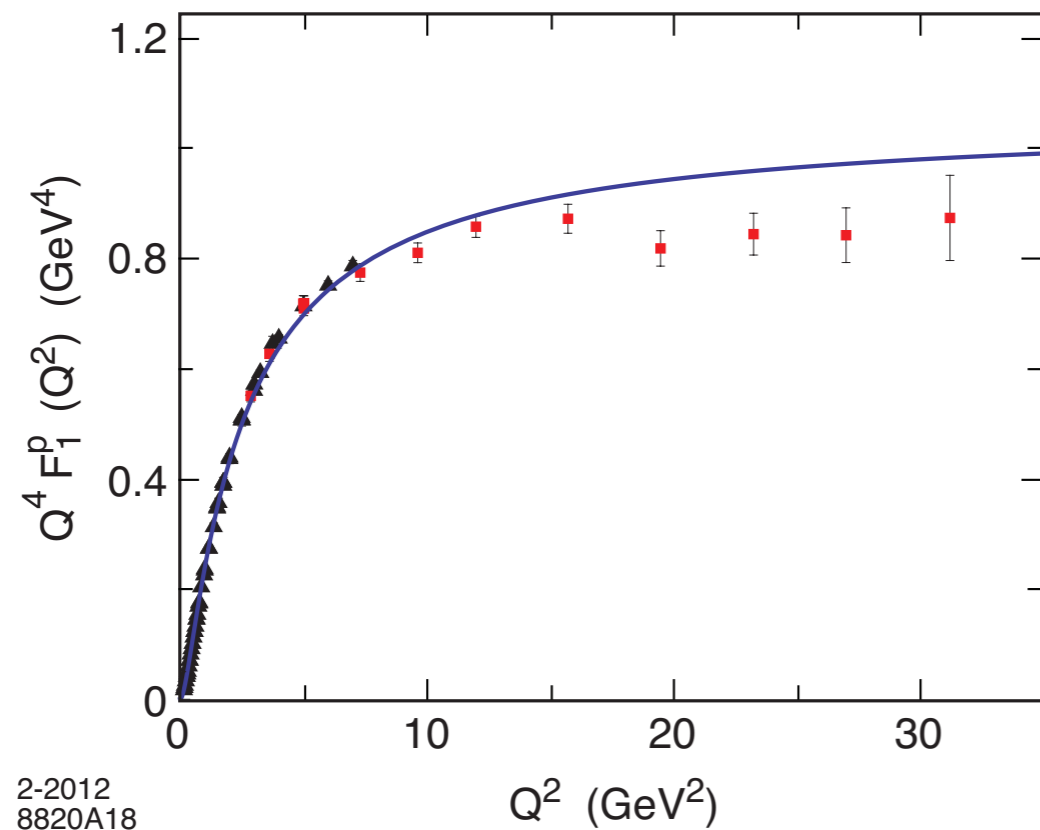
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

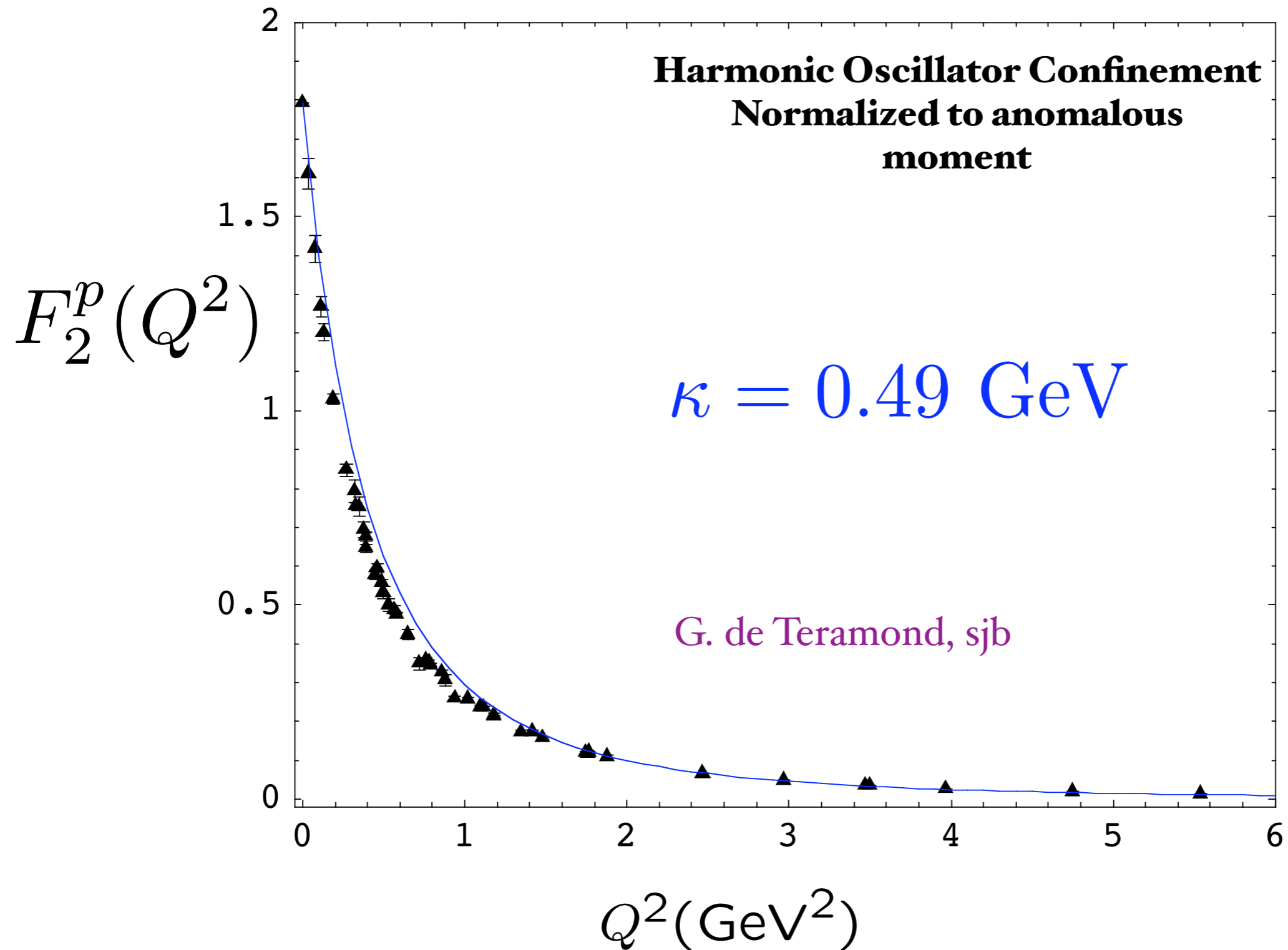
where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

Using $SU(6)$ flavor symmetry and normalization to static quantities



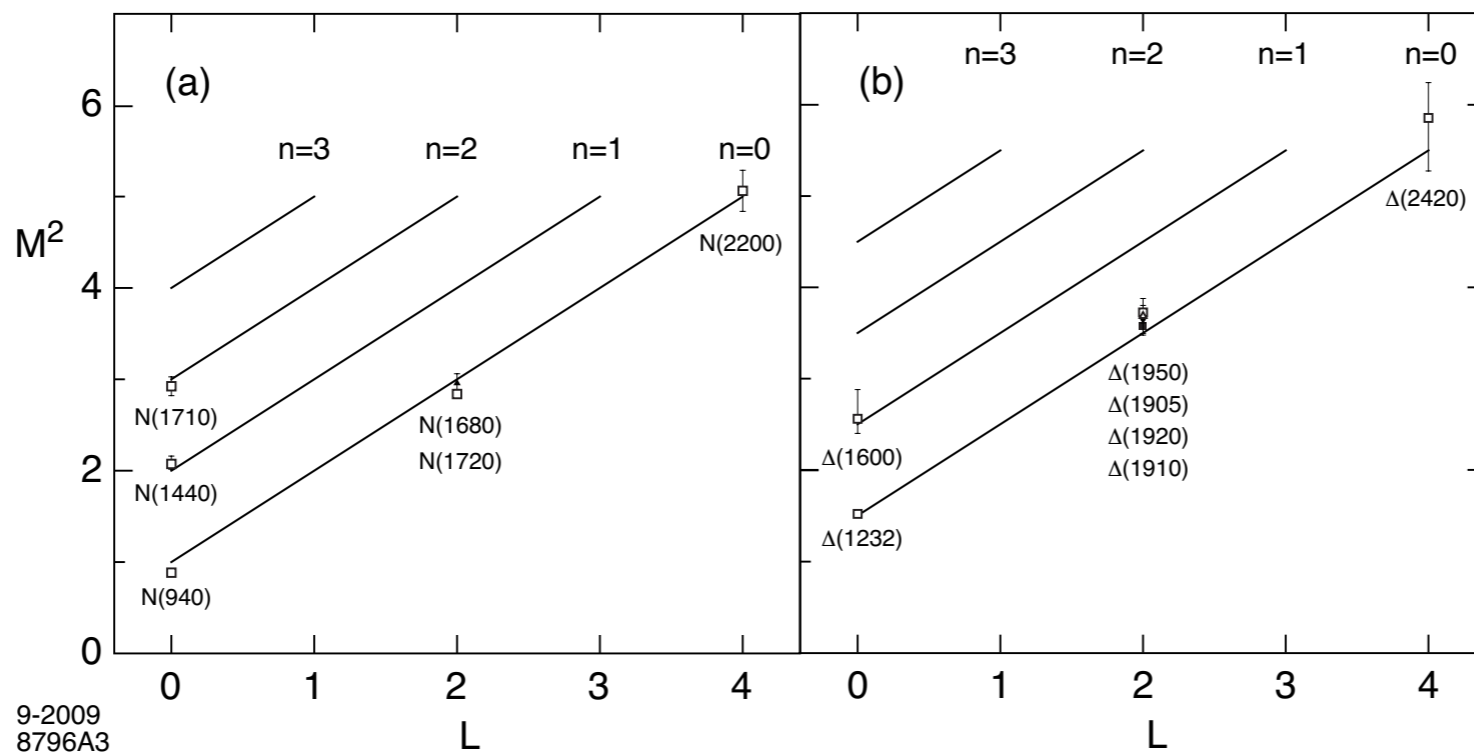
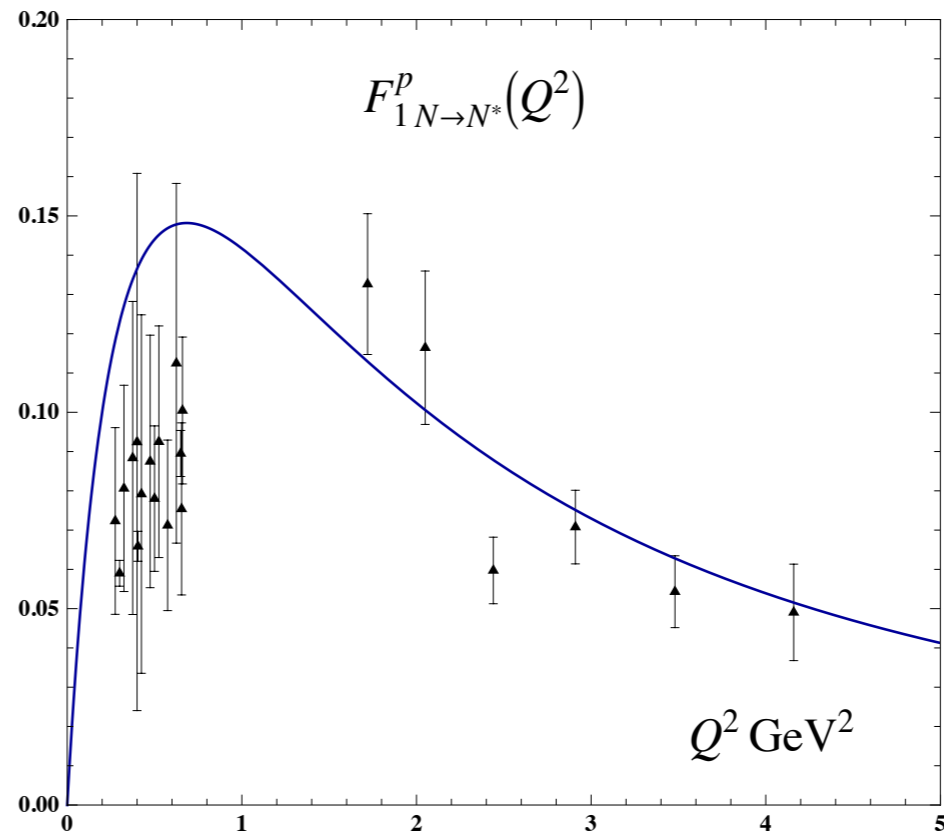
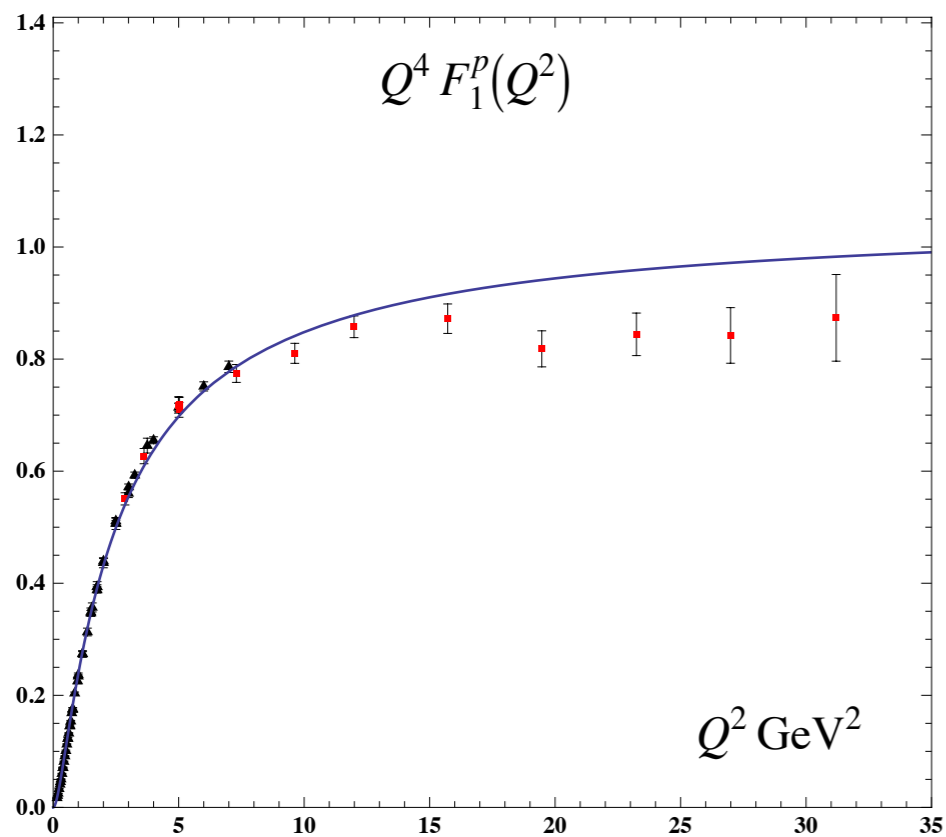
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Excited Baryons in Holographic QCD

G. de Teramond & sjb



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \rightarrow N^*(1440)$: $\Psi_+^{n=0,L=0} \rightarrow \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1N \rightarrow N^*}^p(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q, z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $(F_{1N \rightarrow N^*}^p(0) = 0, \quad V(Q=0, z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

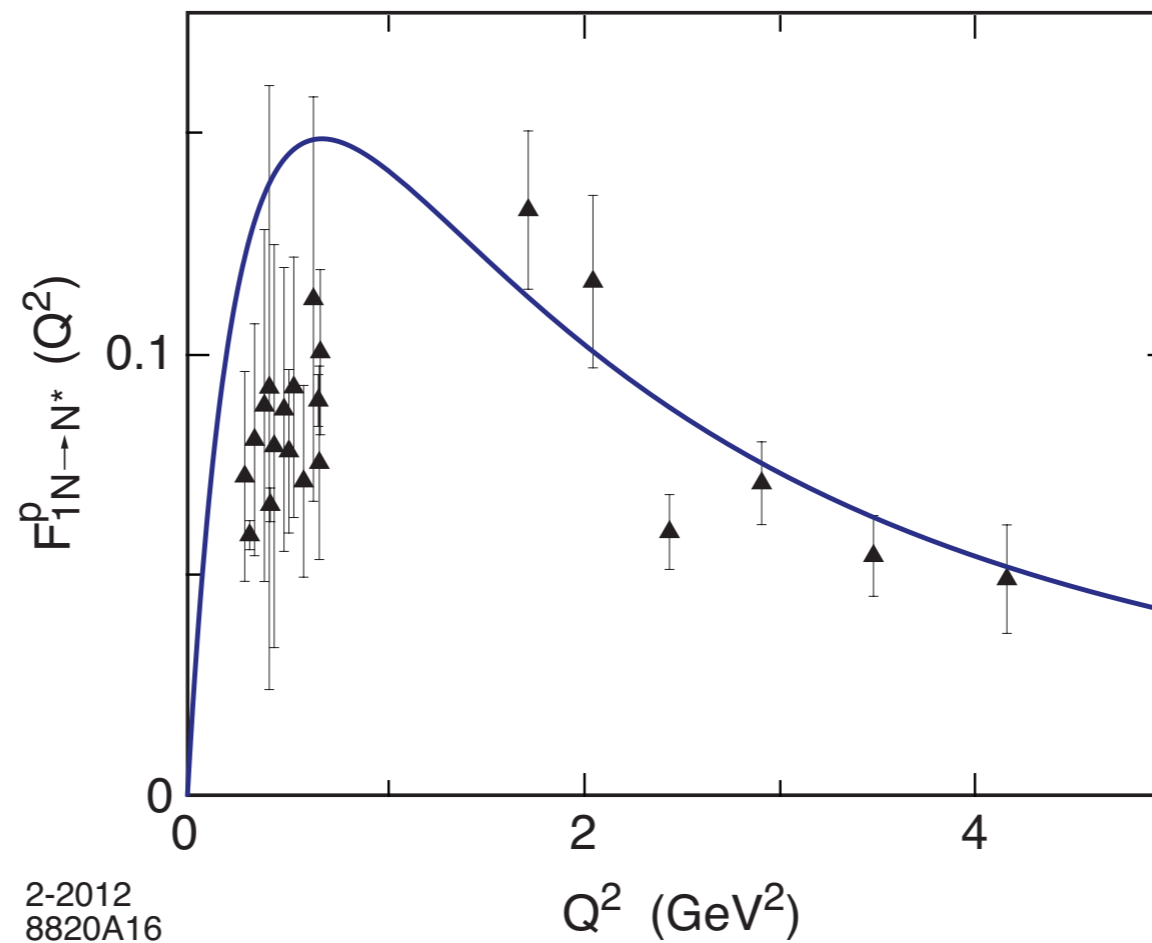
with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$

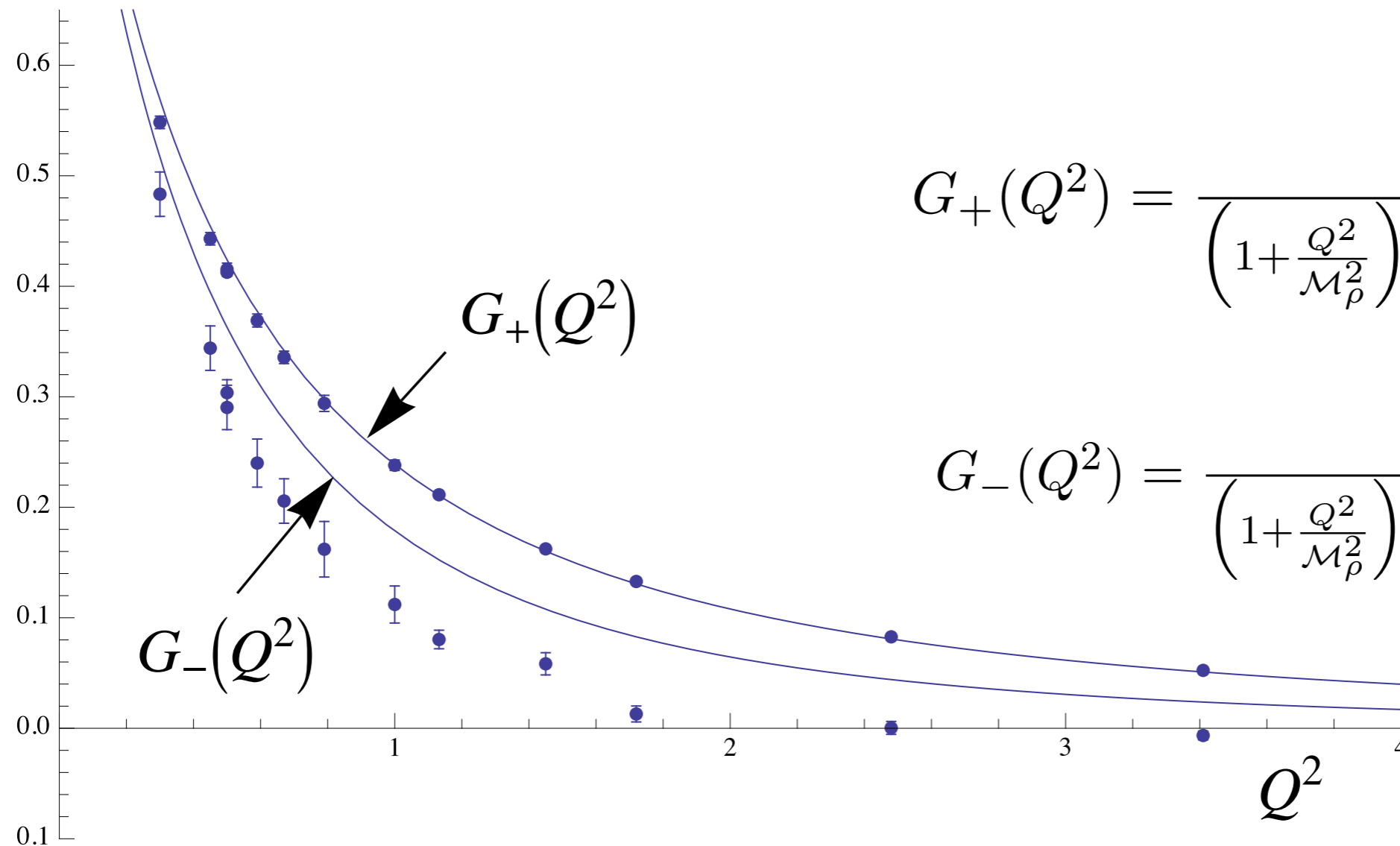


Proton transition form factor to the first radial excited state. Data from JLab

Flavor Decomposition of Elastic Nucleon Form Factors

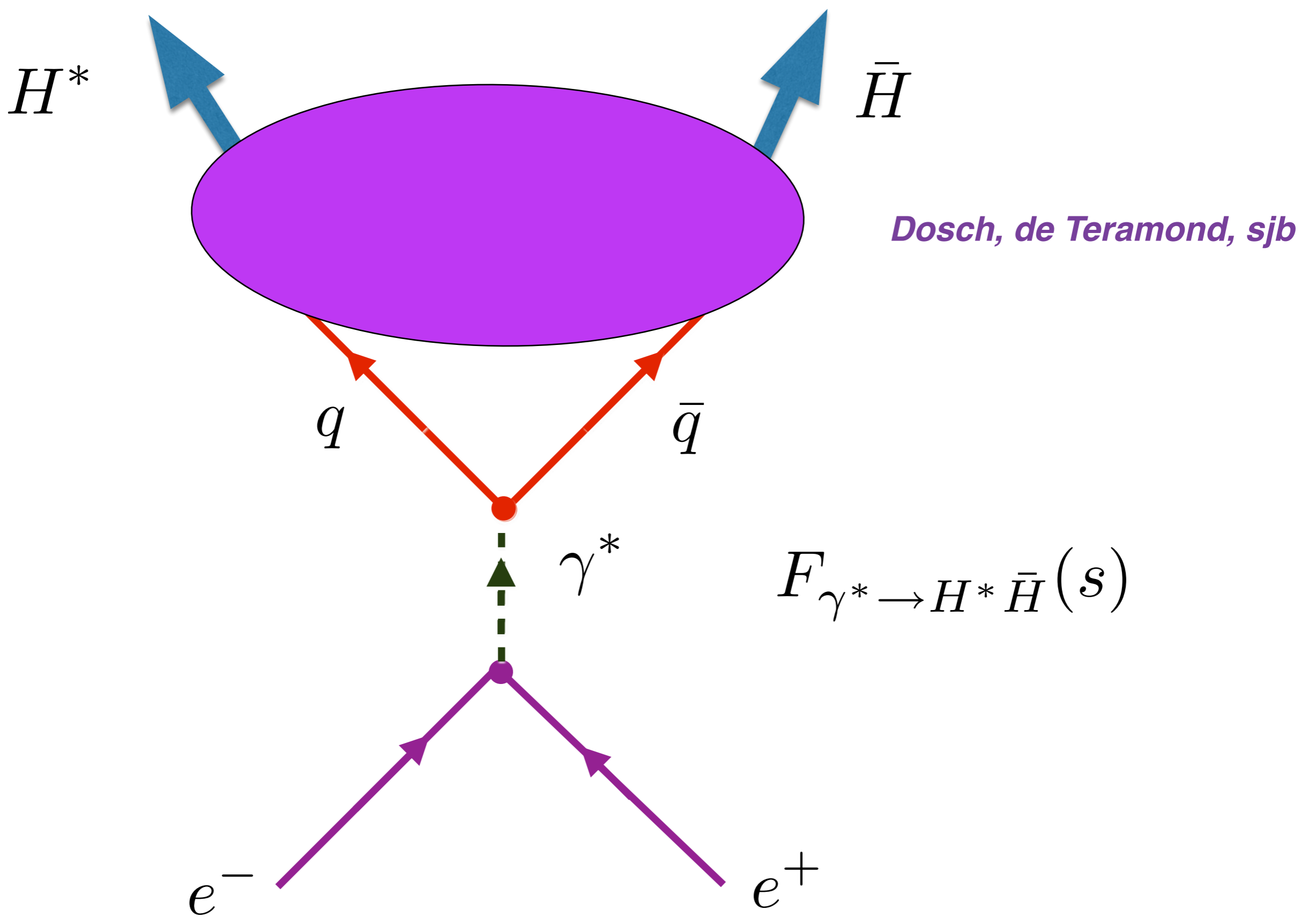
G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F_{u,1}^p = \frac{5}{3}G_+ + \frac{1}{3}G_-$, $F_{d,1}^p = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F_{u,1}^n = \frac{1}{3}G_+ + \frac{2}{3}G_-$, $F_{d,1}^n = \frac{5}{3}G_+ + \frac{1}{3}G_-$



$$G_+(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right)}$$

$$G_-(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \left(1 + \frac{Q^2}{M_{\rho''}^2}\right)}$$



Prediction from Super Conformal AdS/QCD:
 Same Form Factors for $H=M$ and $H=B$ if $L_M=L_B+1$

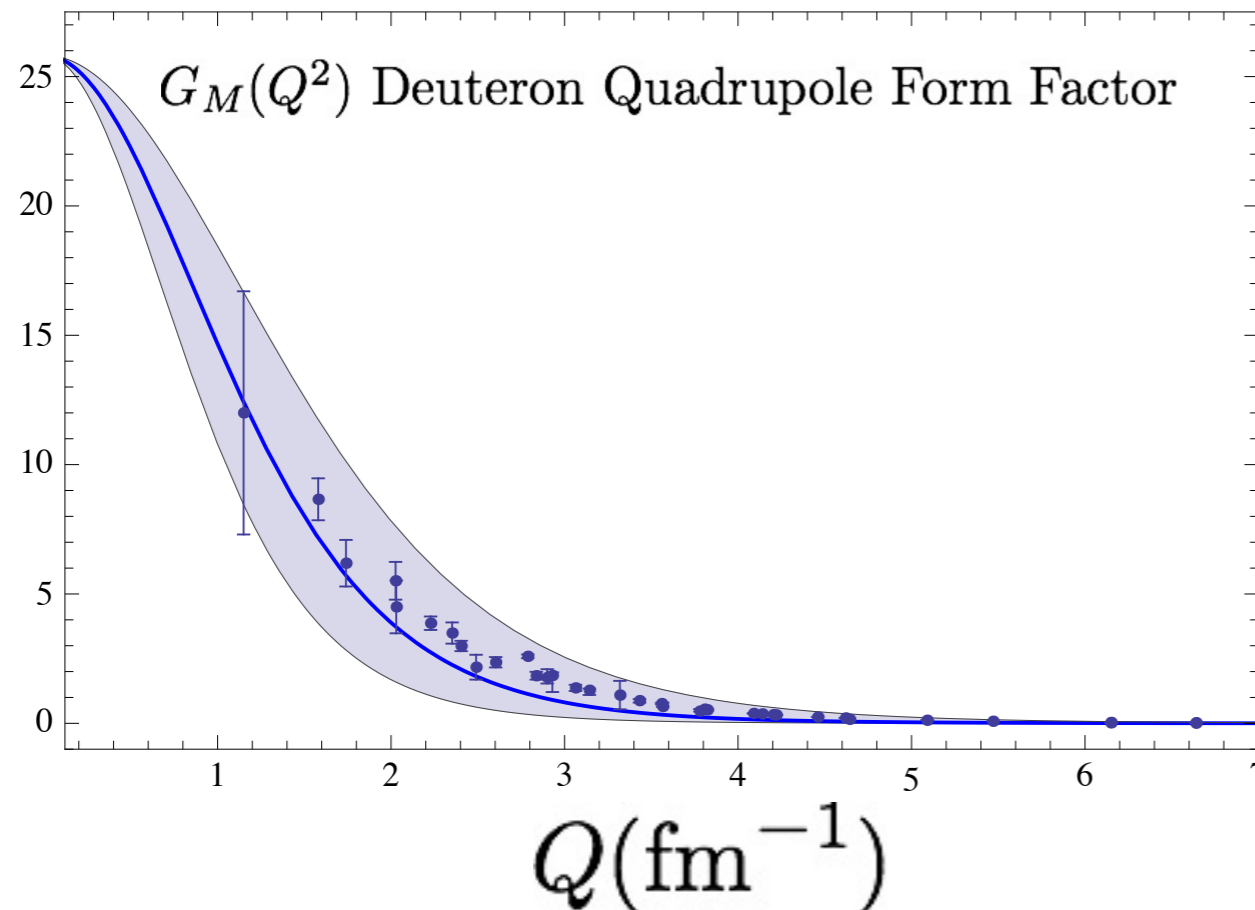
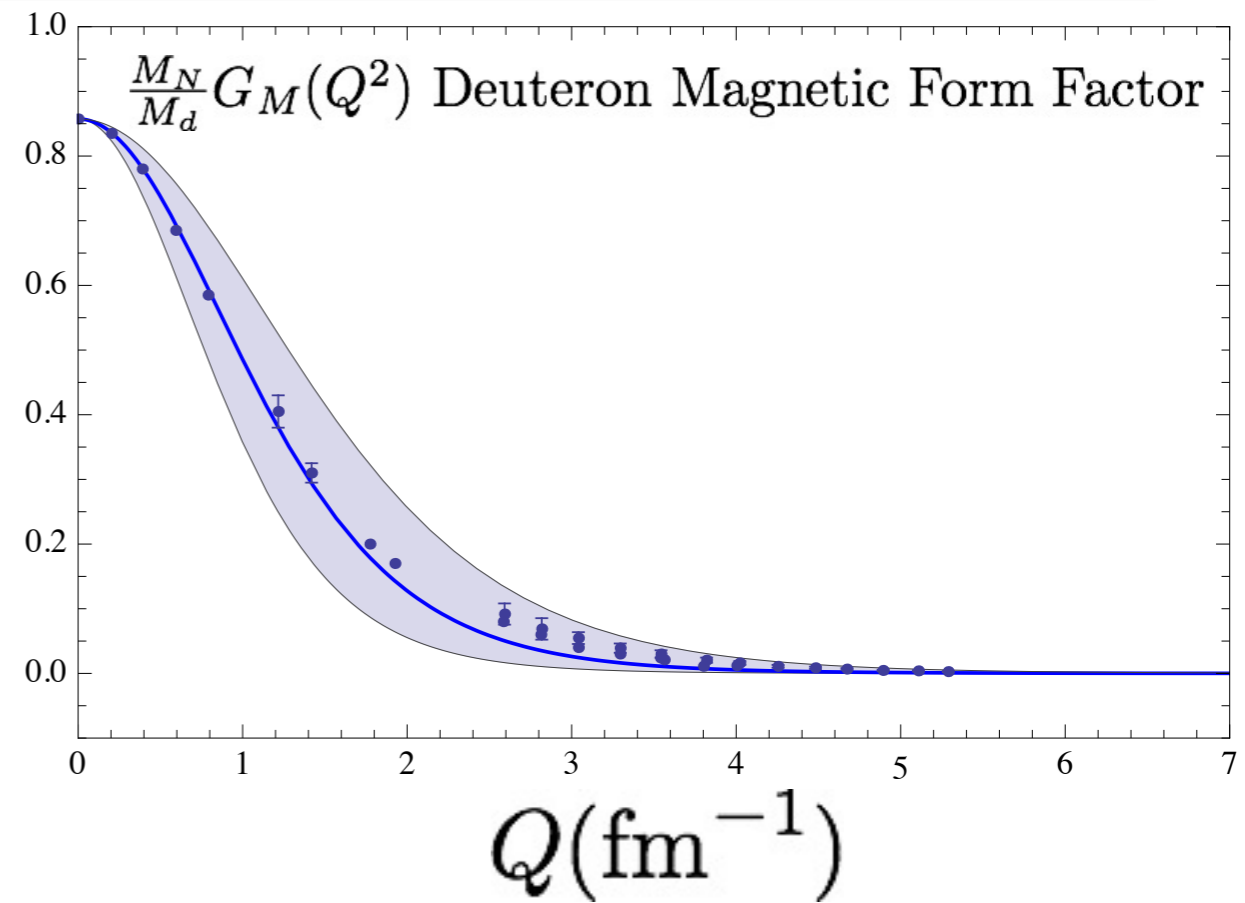
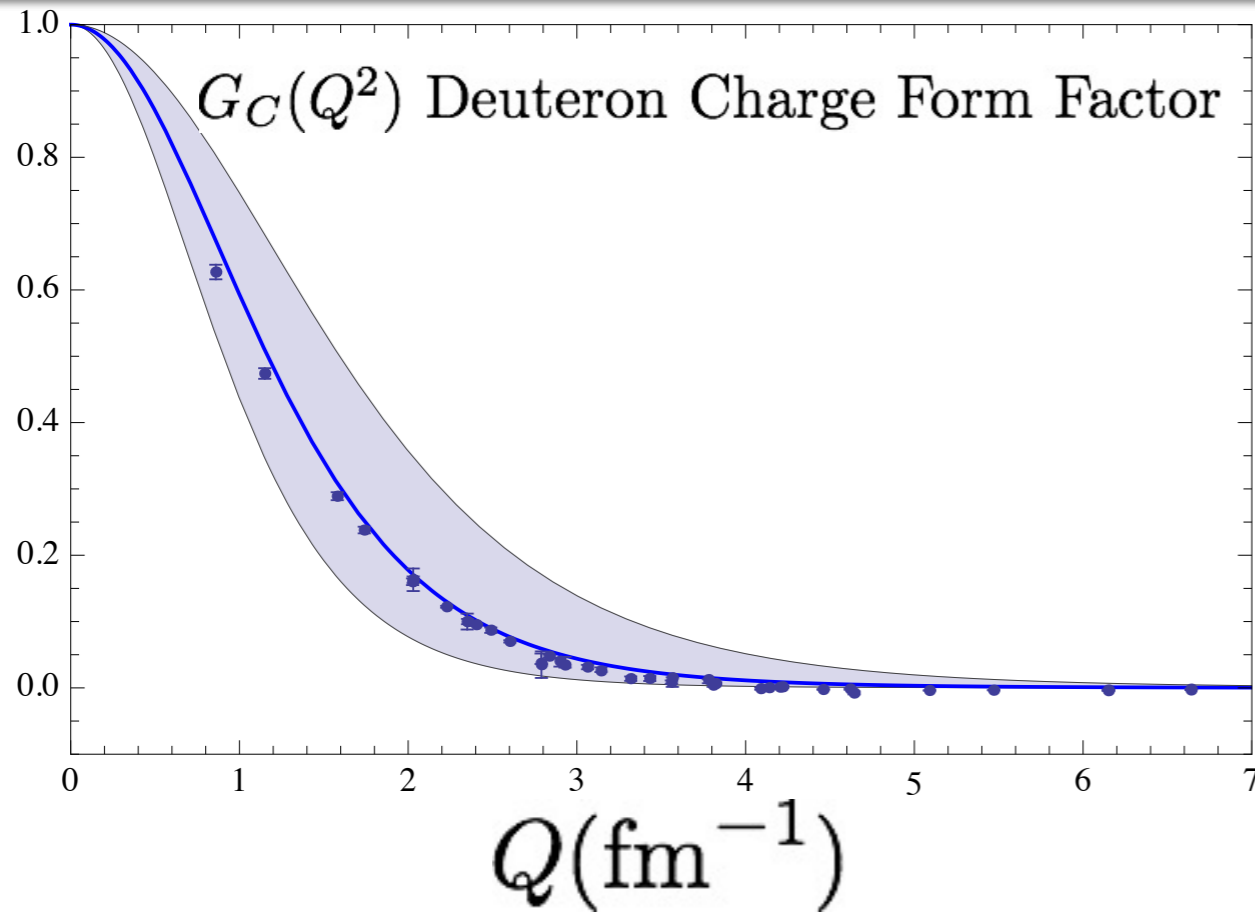
Nuclear physics in soft-wall AdS/QCD: Deuteron electromagnetic form factors

Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt, Alfredo Vega

We present a high-quality description of the deuteron electromagnetic form factors in a soft-wall AdS/QCD approach. We first propose an effective action describing the dynamics of the deuteron in the presence of an external vector field. Based on this action the deuteron electromagnetic form factors are calculated, displaying the correct $1/Q^{10}$ power scaling for large Q^2 values. This finding is consistent with quark counting rules and the earlier observation that this result holds in confining gauge/gravity duals. The Q^2 dependence of the deuteron form factors is defined by a single and universal scale parameter κ , which is fixed from data.

arXiv:1501.02738 [hep-ph]

Application of Light-Front Holography to the Deuteron Form Factors



Thomas Gutsche, Valery E. Lyubovitskij,
Ivan Schmidt, and Alfredo Vega

<http://arxiv.org/abs/1501.02738v3>

Consistent with quark counting rules
Ji, Lepage, sjb

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge

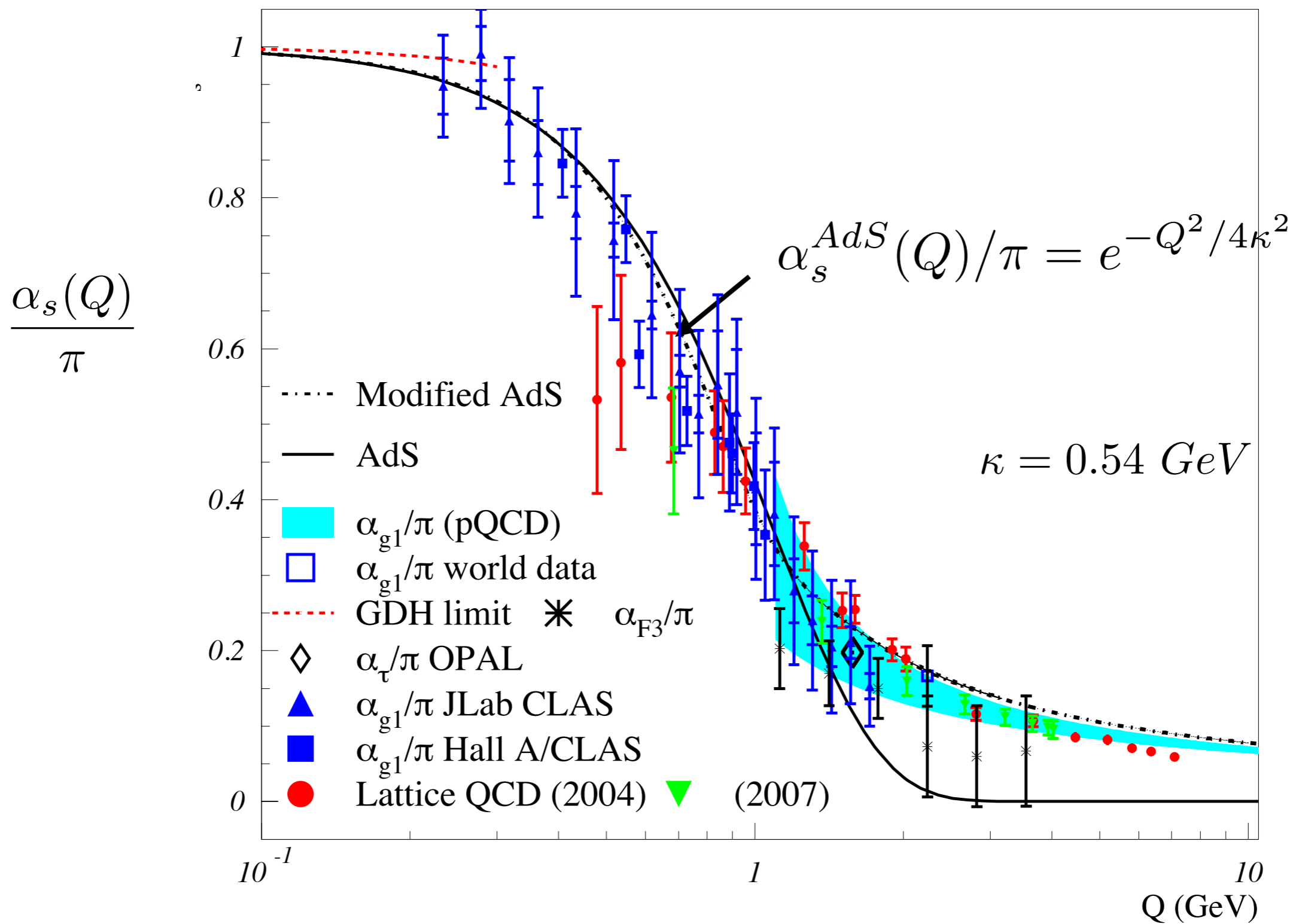
$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large Q^2**
- **Computable at large Q^2 in any pQCD scheme**
- **Universal β_0, β_1**



Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for $Q < 1 \text{ GeV}$

$$e^\varphi = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa_1$$

Deur, de Teramond, sjb

All-Scale QCD Coupling

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$

$$e^{-\frac{Q^2}{4\kappa^2}}$$

Expt:
 $\Lambda_{\overline{MS}} = 0.341 \pm 0.024 \text{ GeV}$

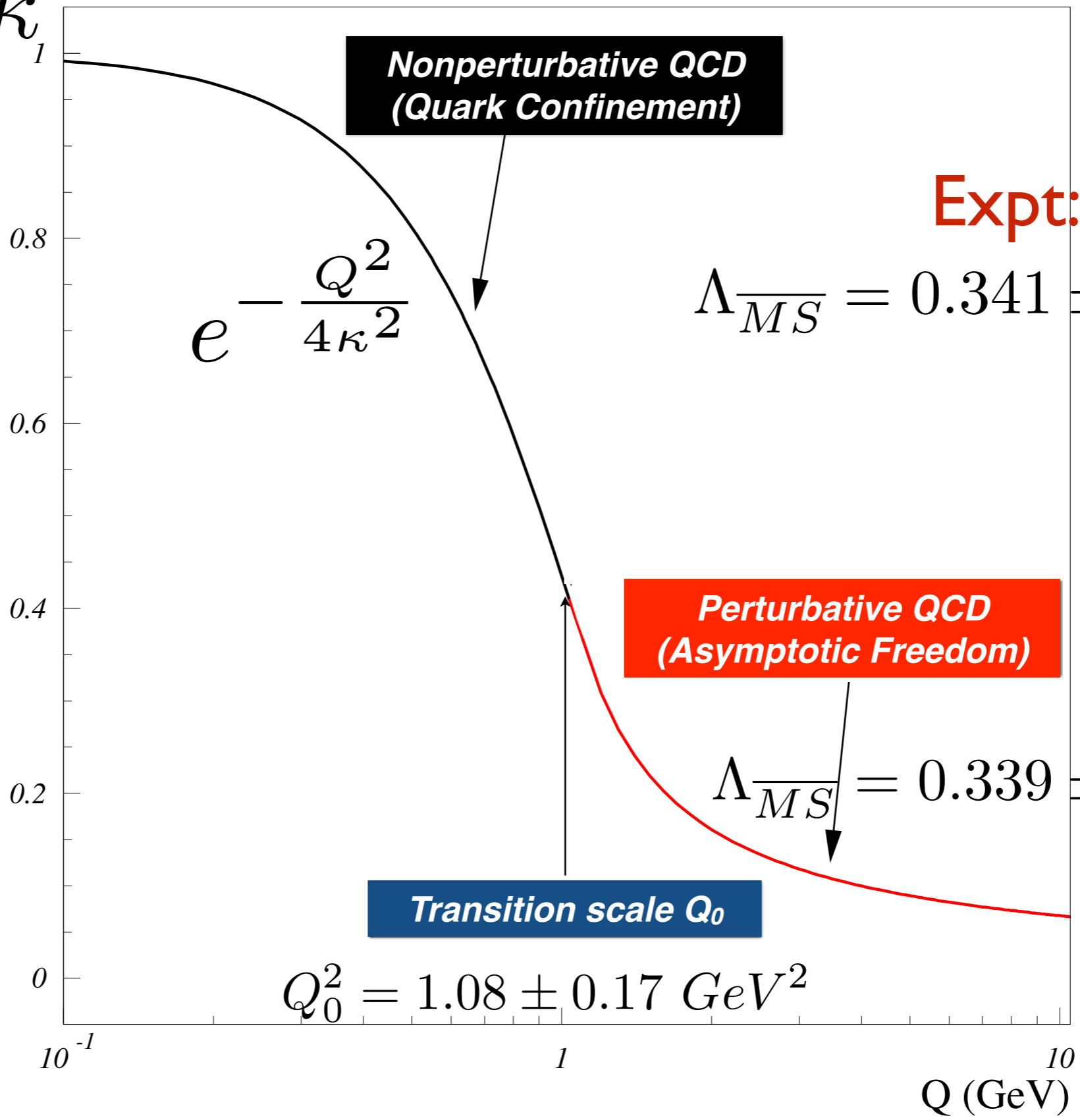
**Perturbative QCD
(Asymptotic Freedom)**

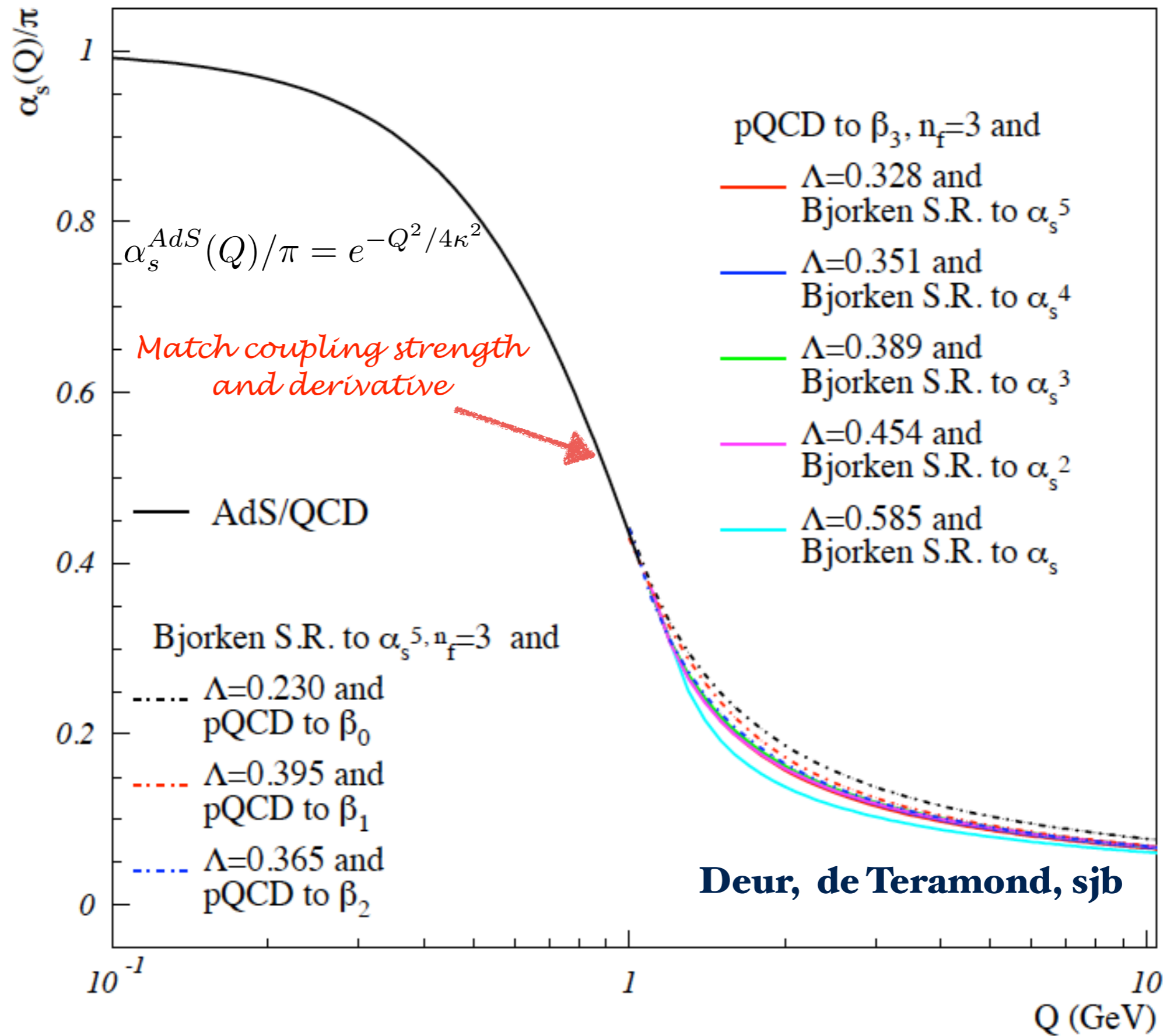
$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016 \text{ GeV}$$

Transition scale Q_0

$$Q_0^2 = 1.08 \pm 0.17 \text{ GeV}^2$$

$$\lambda \equiv \kappa^2$$





$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa_1$$

Deur, de Teramond, sjb

All-Scale QCD Coupling

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$

$$e^{-\frac{Q^2}{4\kappa^2}}$$

**Nonperturbative QCD
(Quark Confinement)**

**Cloet, Roberts:
No quark mass pole in DSE**

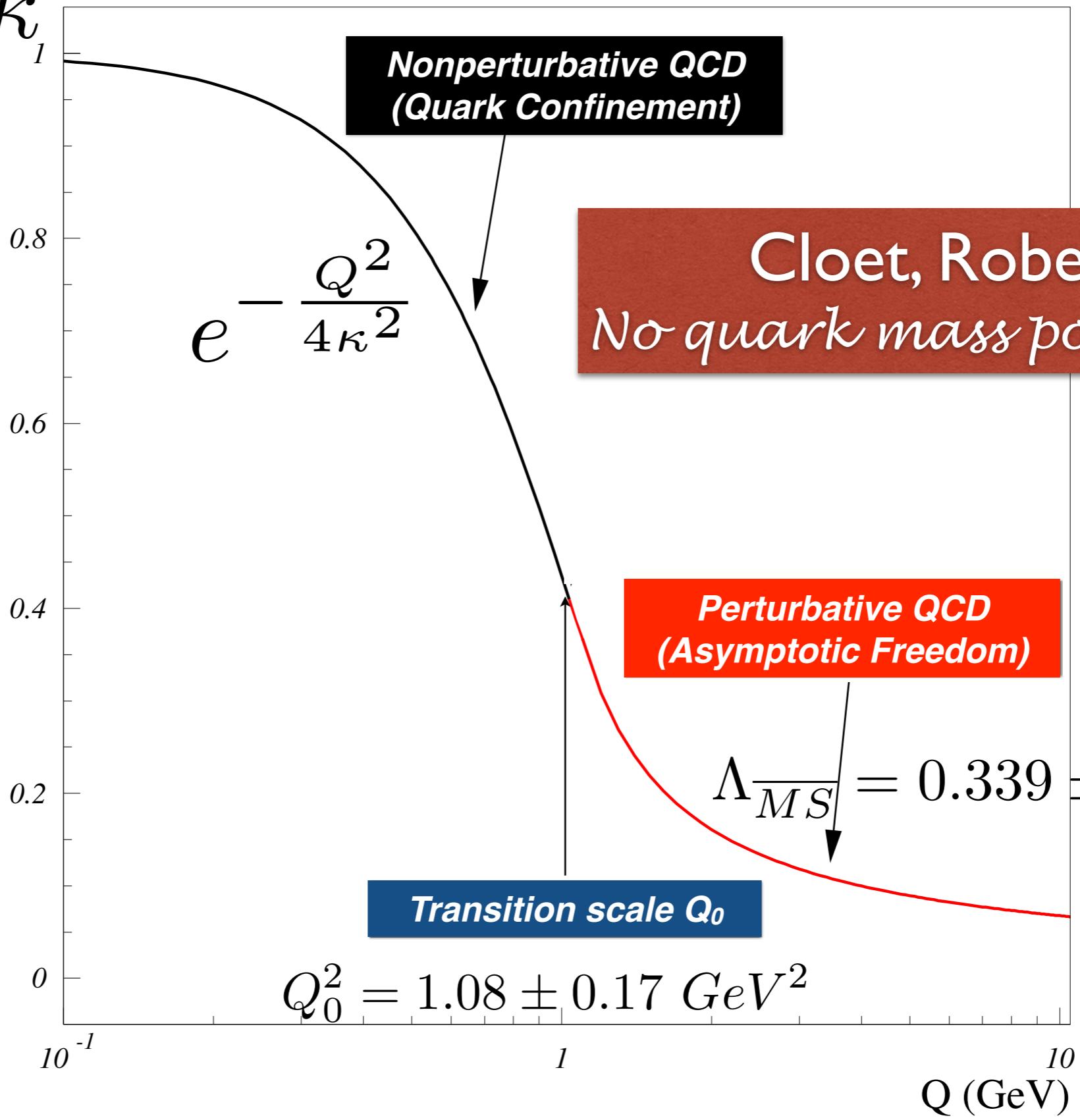
**Perturbative QCD
(Asymptotic Freedom)**

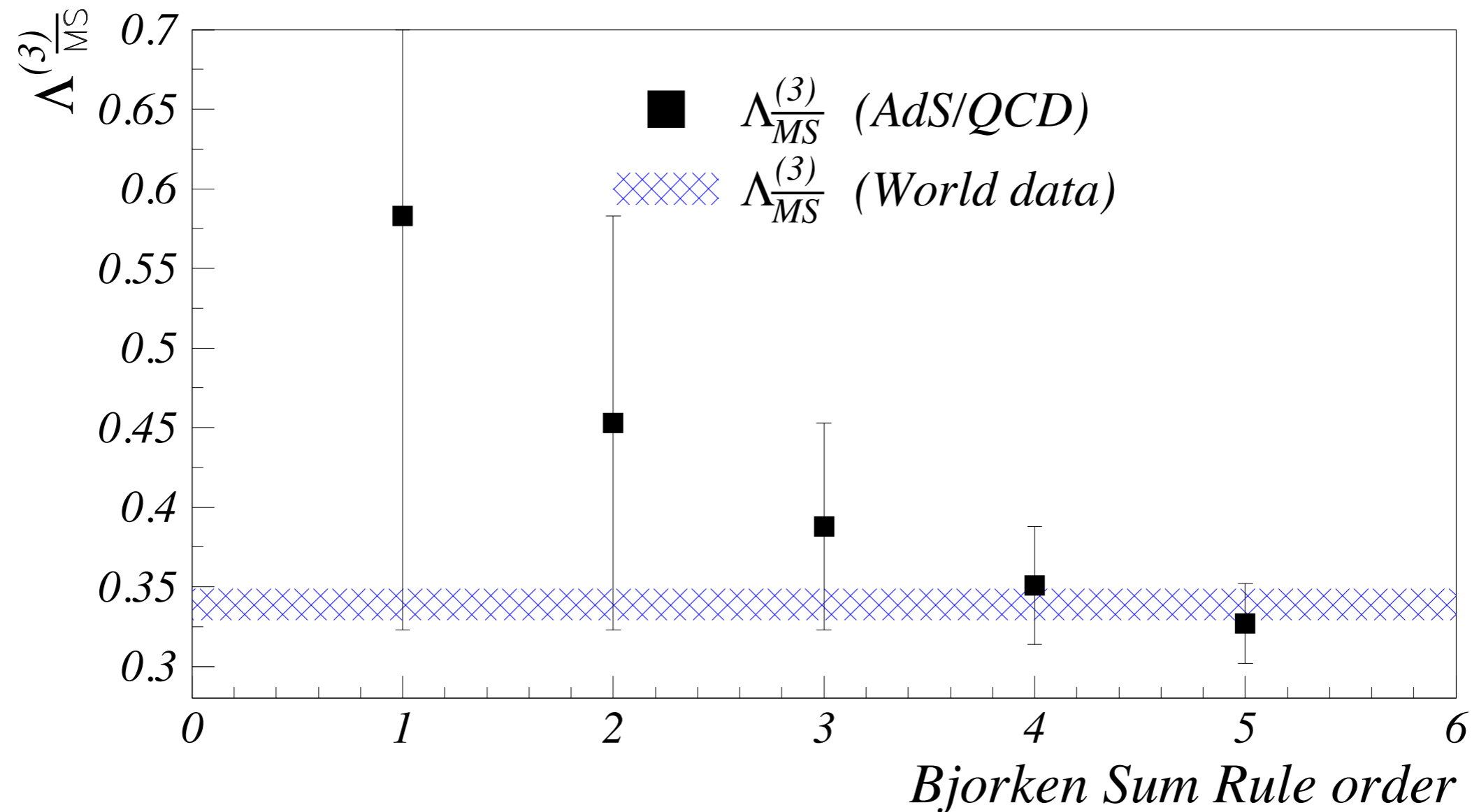
$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016 \text{ GeV}$$

Transition scale Q_0

$$Q_0^2 = 1.08 \pm 0.17 \text{ GeV}^2$$

$$\lambda \equiv \kappa^2$$





$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983 \frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328 \text{ GeV}$$

Connect $\Lambda_{\overline{MS}}$ to hadron masses!

Experiment: $M_\rho = 0.7753 \pm 0.0003 \text{ GeV}$

Interpretation of Mass Scale \mathcal{K}

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of \mathcal{K}
- Value of \mathcal{K} itself not determined -- place holder
- Need external constraint such as f_π

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks



Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

Connection to the Linear Instant-Form Potential

- Compare invariant mass in the instant-form in the hadron center-of-mass system $\mathbf{P} = 0$,

$$M_{q\bar{q}}^2 = 4m_q^2 + 4\mathbf{p}^2$$

with the invariant mass in the front-form in the constituent rest frame, $\mathbf{k}_q + \mathbf{k}_{\bar{q}} = 0$

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_{\perp}^2 + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2\sqrt{\mathbf{p}^2 + m_q^2}V + 2V\sqrt{\mathbf{p}^2 + m_q^2}$$

where $\mathbf{p}_{\perp}^2 = \frac{\mathbf{k}_{\perp}^2}{4x(1-x)}$, $p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and V is the effective potential in the instant-form

- For small quark masses a linear instant-form potential V implies a harmonic front-form potential U and thus linear Regge trajectories

AdS/QCD and Light-Front Holography

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

- **Zero mass pion for $m_q = 0$ ($n=J=L=0$)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2 : Dimensional counting**
 $[Q^2]^{n-1} F(Q^2) \rightarrow \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$



Features of AdS/QCD

- **Color confining potential $\kappa^4 \zeta^2$ and universal mass scale from dilaton**
$$e^{\phi(z)} = e^{\kappa^2 z^2} \quad \alpha_s(Q^2) \propto \exp -Q^2/4\kappa^2$$
- **Dimensional transmutation** $\Lambda_{\overline{MS}} \leftrightarrow \kappa \leftrightarrow m_H$
- **Chiral Action remains conformally invariant despite mass scale** *DAFF*
- **Light-Front Holography: Duality of AdS and frame-independent LF QCD**
- **Reproduces observed Regge spectroscopy — same slope in n, L, and J for mesons and baryons**
- **Massless pion for massless quark**
- **Supersymmetric meson-baryon dynamics and spectroscopy:**
 $L_M = L_B + I$
- **Dynamics: LFWFs, Form Factors, GPDs**

*Superconformal Algebra
Fubini and Rabinovici*

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

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July 2015



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String Theory

- **Conformal template:**
- **Use isometries of AdS₅**

Goal: First Approximant to QCD

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS₅ space

*Counting rules for Hard Exclusive Scattering
Regge Trajectories*

AdS/QCD

Conformal behavior at short distances

QCD at the Amplitude Level

*Confinement at large distance
Unique!*

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J=0, 1, 1/2, 3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

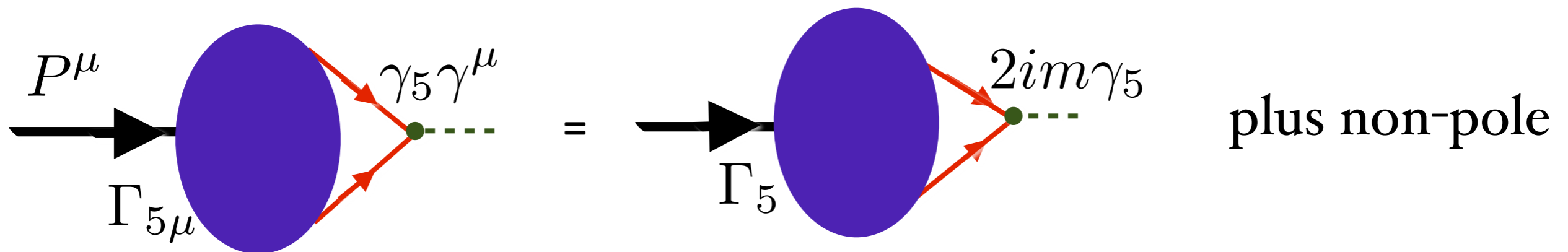
Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffraction dissociation of pion and proton to jets**
- **Identify the factorization Scale for ERBL, DGLAP evolution: Q_0**
- **Compute Tetraquark Spectroscopy Sequentially**
- **Update $SU(6)$ spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization**

Ward-Takahashi Identity for axial current

$$P^\mu \Gamma_{5\mu}(k, P) + 2im\Gamma_5(k, P) = S^{-1}(k + P/2)i\gamma_5 + i\gamma_5 S^{-1}(k - P/2)$$

$$S^{-1}(\ell) = i\gamma \cdot \ell A(\ell^2) + B(\ell^2) \quad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



Identify pion pole at $P^2 = m_\pi^2$

$$P^\mu \langle 0 | \bar{q} \gamma_5 \gamma^\mu q | \pi \rangle = 2m \langle 0 | \bar{q} i \gamma_5 q | \pi \rangle$$

$$f_\pi m_\pi^2 = -(m_u + m_d) \rho_\pi$$

Revised Gell Mann-Oakes-Renner Formula in QCD

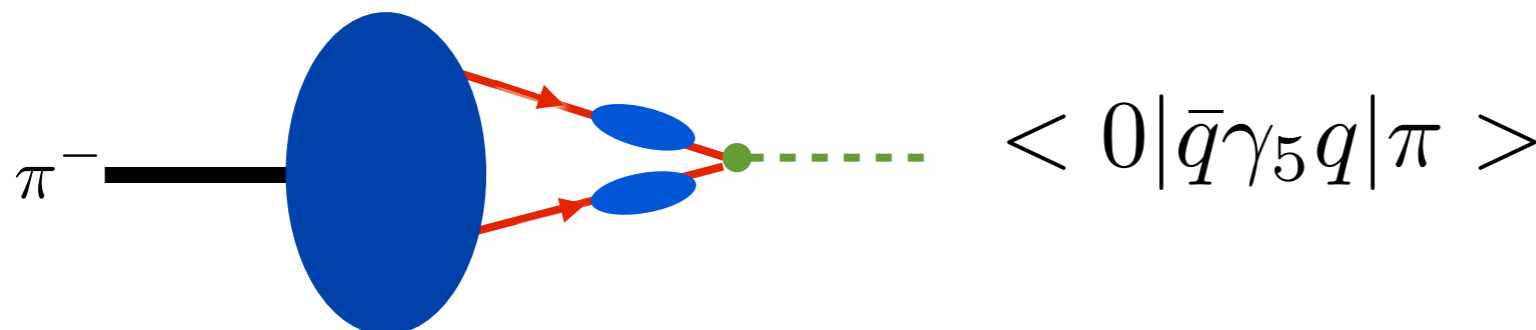
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} \langle 0 | \bar{q}q | 0 \rangle$$

**current algebra:
effective pion field**

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} \langle 0 | i\bar{q}\gamma_5 q | \pi \rangle$$

**QCD: composite pion
Bethe-Salpeter Eq.**

vacuum condensate actually is an "in-hadron condensate"



Maris, Roberts, Tandy

Standard Model on the Light-Front

- Same phenomenological predictions
- Higgs field has three components
- Real part creates Higgs particle
- Imaginary part (Goldstone) become longitudinal components of W, Z
- *Higgs VEV of instant form becomes $k^+=0$ LF zero mode!*
- Analogous to a background static classical Zeeman or Stark Fields
- Zero contribution to T^{μ}_{μ} ; zero coupling to gravity



Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0\rangle = E_0|\psi_0\rangle, E_0 = \min\{E_i\}$$

*Eigenstate defined at one time t over all space;
Acausal! Frame-Dependent*

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

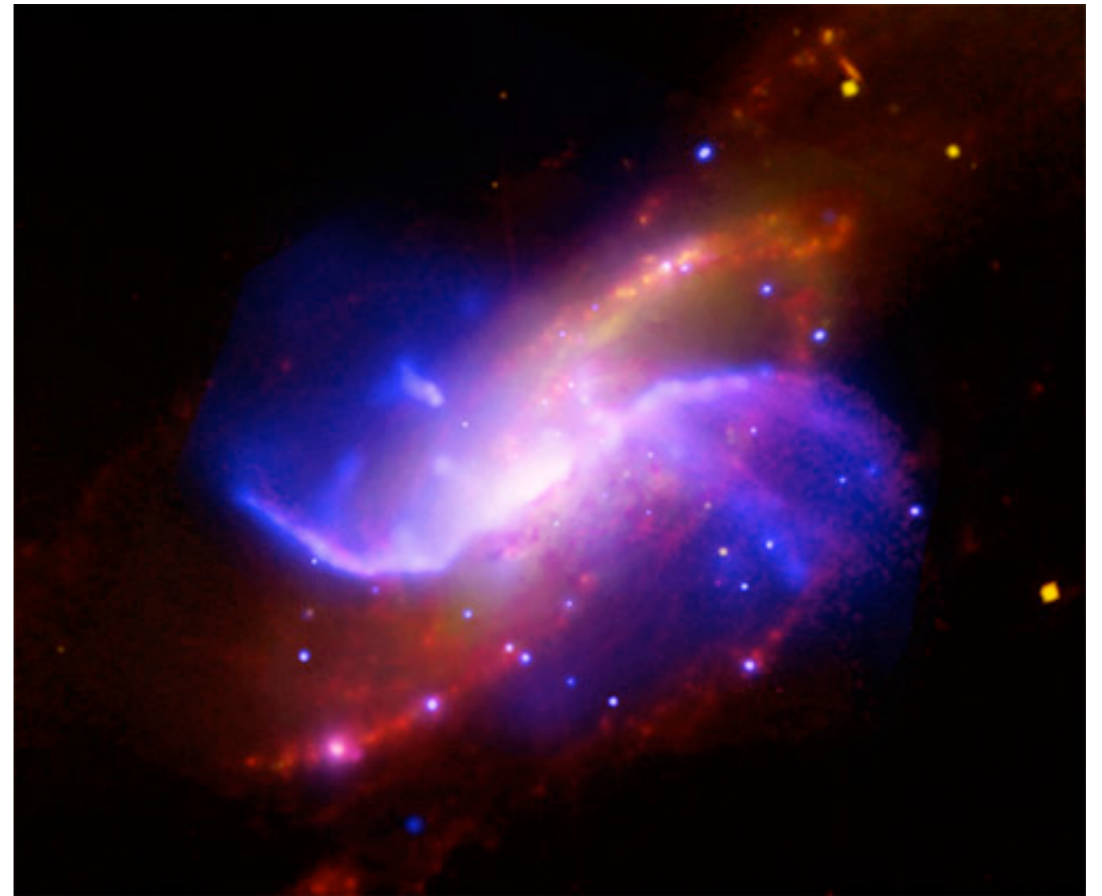
$$H_{LF}|\psi_0\rangle_{LF} = M_0^2|\psi_0\rangle_{LF}, M_0^2 = 0.$$

*Frame-independent eigenstate at fixed LF time $\tau = t+z/c$
within causal horizon*

Frame-independent description of the causal physical universe!

*We view the universe
as light reaches us
along the light-front
at fixed*

$$\tau = t + z/c$$



Front Form Vacuum Describes the Empty, Causal Universe

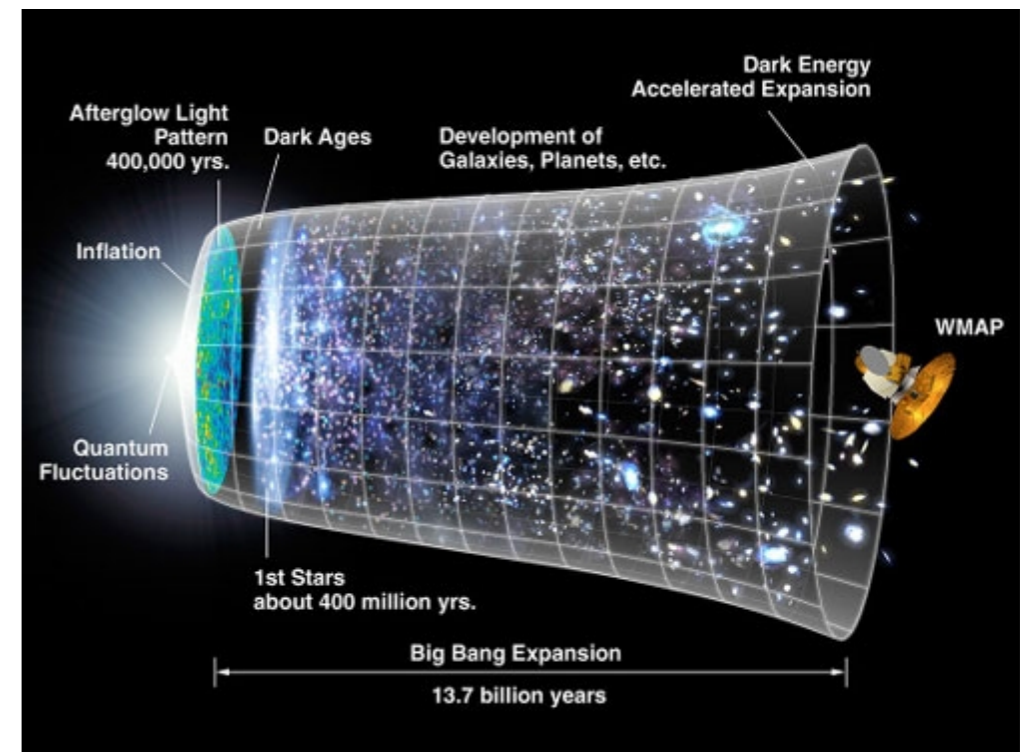
Front Form Vacuum Describes the Empty, Causal Universe

- $P^+ = \sum_i p_i^+$, $p_i^+ > 0$: LF vacuum is the state with $P^+ = 0$ and contains no particles: all other states have $P^+ > 0$ (usual vacuum bubbles are kinematically forbidden in the front form !)
- Frame independent definition of the vacuum within the causal horizon

$$P^2|0\rangle = 0$$

(LF vacuum also has zero quantum numbers and $P^+ = 0$)

- LF vacuum is defined at fixed LF time $x^+ = x^0 + x^3$ over all $x^- = x^0 - x^3$ and \mathbf{x}_\perp , the expanse of space that can be observed within the speed of light
- Causality is maintained since LF vacuum only requires information within the causal horizon
- The front form is a natural basis for cosmology: universe observed along the front of a light wave



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Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state $M=0$.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: “In-hadron” condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD

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Light-front formulation of the standard model

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Light-front (LF) quantization in the light-cone (LC) gauge is used to construct a renormalizable theory of the standard model. The framework derived earlier for QCD is extended to the Glashow-Weinberg-Salam (GWS) model of electroweak interaction theory. The Lorentz condition is automatically satisfied in LF-quantized QCD in the LC gauge for the free massless gauge field. In the GWS model, with the spontaneous symmetry breaking present, we find that the 't Hooft condition accompanies the LC gauge condition corresponding to the massive vector boson. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD. The nontransverse and linearly independent third polarization vector is found to be parallel to the gauge direction. The corresponding sum over polarizations in the standard model, indicated by $K_{\mu\nu}(k)$, has several simplifying properties similar to the polarization sum $D_{\mu\nu}(k)$ in QCD. The framework is unitary and ghost free (except for the ghosts at $k^+ = 0$ associated with the light-cone gauge prescription). The massive gauge field propagator has well-behaved asymptotic behavior. The interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, plus additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the Goldstone boson (or electroweak) equivalence theorem, as the illustrations show.

Abelian U(1) LF Model with Spontaneous Symmetry Breaking

$$\mathcal{L} = \partial_+ \phi^\dagger \partial_- \phi + \partial_- \phi^\dagger \partial_+ \phi - \partial_\perp \phi^\dagger \partial_\perp \phi - \mathcal{V}(\phi^\dagger \phi)$$

where $V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$ with $\lambda > 0$, $\mu^2 < 0$

Constraint equation: $\int d^2 x_\perp dx^- [\partial_\perp \partial_\perp \phi - \frac{\delta V}{\delta \phi^\dagger}] = 0$

$$\phi(\tau, x^-, x_\perp) = \omega(\tau, x_\perp) + \varphi(\tau, x^-, x_\perp)$$

$\omega(\tau, x_\perp)$ is a $k^+ = 0$ zero mode

$$\omega = v/\sqrt{2} \text{ where } v = \sqrt{-\mu^2/\lambda}$$

Thus a c-number in LF replaces conventional Higgs VEV

No coupling to gravity!

Possibility: $\partial_\perp \omega \neq 0$

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(\text{expt})$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:

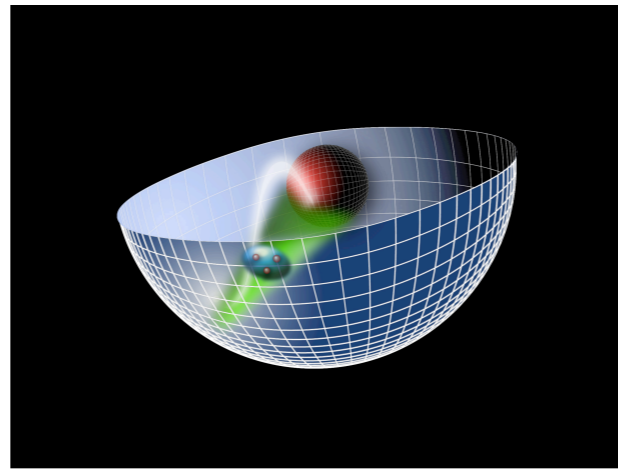
(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD “Condensates”

(C) Higgs Light-Front Zero Mode

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

***Unique
Confinement Potential!***

*Preserves Conformal Symmetry
of the action*

$$\kappa \simeq 0.6 \text{ GeV}$$

Confinement scale:

$$1/\kappa \simeq 1/3 \text{ fm}$$

● **de Alfaro, Fubini, Furlan:**

● **Fubini, Rabinovici:**

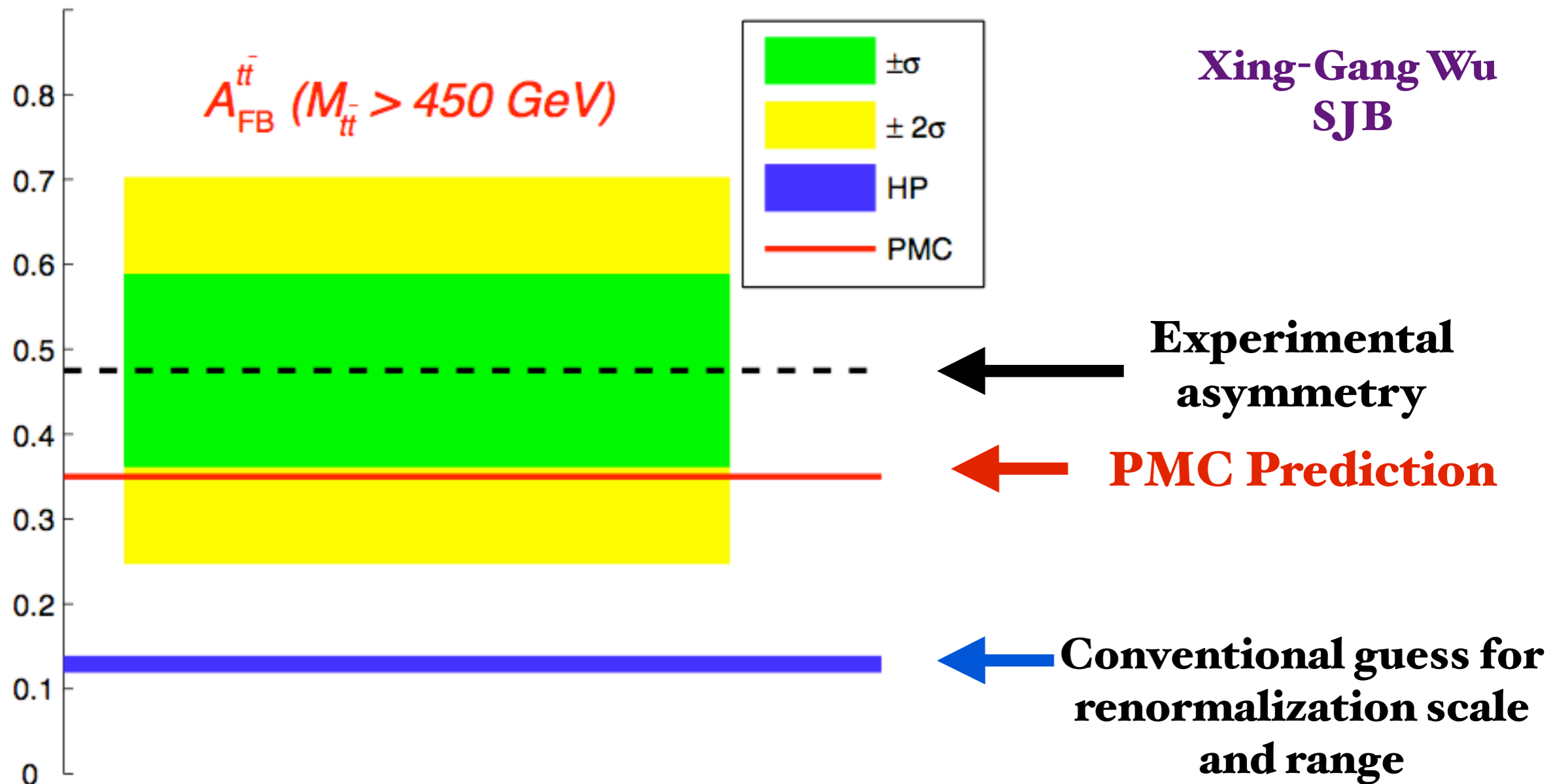
***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!***

QCD Myths

- **Anti-Shadowing is Universal**
- **ISI and FSI are higher twist effects and universal**
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **Heavy quarks only from gluon splitting**
- **Renormalization scale cannot be fixed**
- **QCD condensates are vacuum effects**
- **QCD gives 10^{42} to the cosmological constant**



The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)

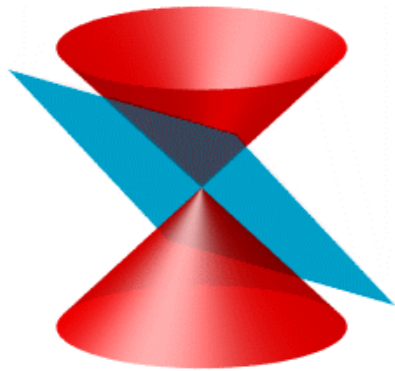
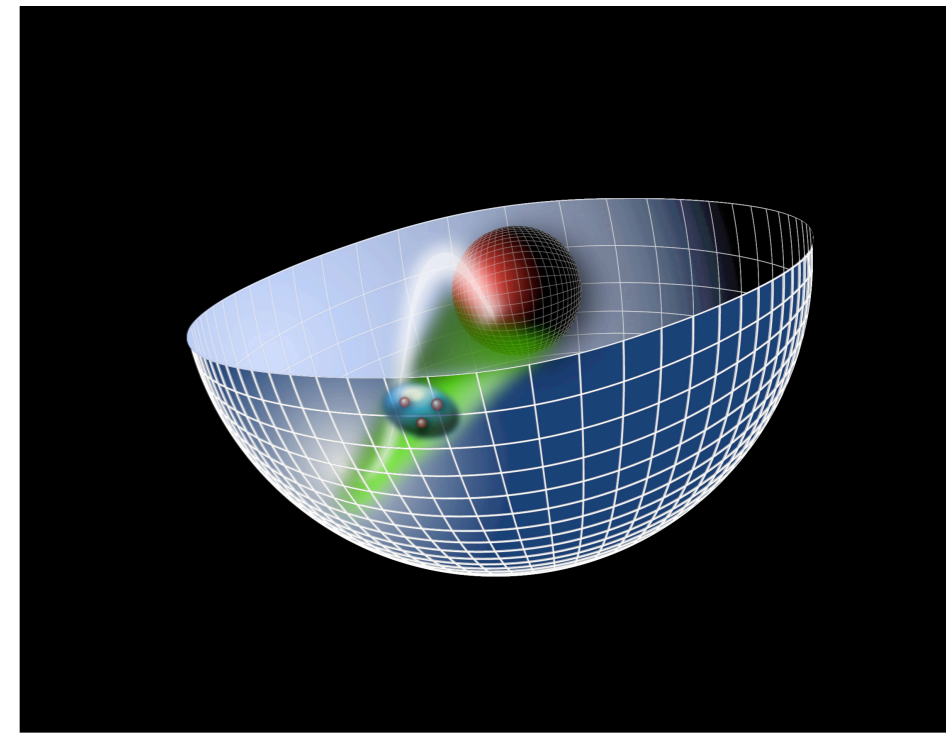
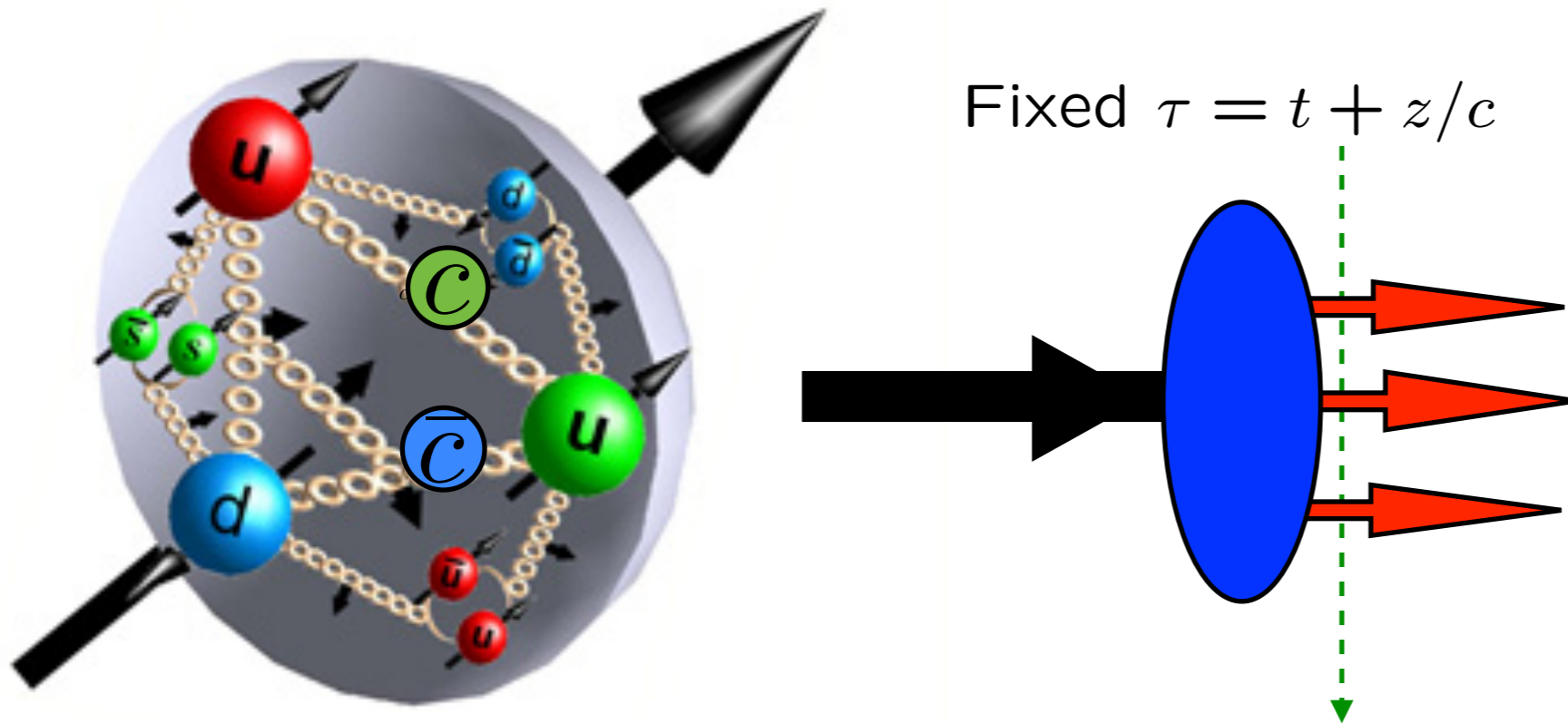


Top quark forward-backward asymmetry predicted by pQCD NNLO within 1σ of CDF/D0 measurements using PMC/BLM scale setting

Features of BLM/PMC

- **Predictions are scheme-independent**
- **Matches conformal series**
- **Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsmann, sjb)**
- **No $n!$ Renormalon growth**
- **New scale at each order; n_F determined at each order**
- **Multiple Physical Scales Incorporated**
- **Rigorous: Satisfies all Renormalization Group Principles**
- **Realistic Estimate of Higher-Order Terms**
- **Eliminates unnecessary theory error**

New Perspectives for Hadron Physics and the Cosmological Constant Problem



What Comes Beyond the Standard Model?



*Bled, Slovenia
July 17, 2015*

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